Gain Characterization of the RF Measurement Path

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GAIN CHARACTERIZATION OF THE RF MEASUREMENT PATH

J. Wayde Allen *

In radio frequency (RF) measurements the gain (or loss) of the signal path connecting the measurement equipment to the measurement reference plane must be accounted for. This tutorial paper discusses the various definitions of gain, and how to determine the gain using either a calibrated signal generator, noise source, or network analyzer.

Key words: available power; cable loss; calibration; delivered power; gain; mismatch

1.   INTRODUCTION

At the Institute for Telecommunication Sciences (ITS) systems such as the Radio Spectrum Measurement System (RSMS) are often called upon to perform a wide range of radio frequency (RF) measurements. For this reason, the RSMS is not a single system but rather a toolbox of equipment that can be configured in a variety of ways. While this gives the RSMS a great deal of flexibility, it complicates the job of the engineer who must provide an analysis of the inherent measurement uncertainties for a given test. The uncertainties associated with each test configuration need to be dealt with on a case-by-case basis. The goal here is to provide a collection of basic uncertainty analyses to be used as a starting point for determining the overall test uncertainty. In a sense, this means creating a toolbox of uncertainty analysis methods that can be adapted as needed. The measurement configurations and analyses that follow deal specifically with the problem of accounting for the gain or loss in the RF path from where the signal is sampled to where it is ultimately detected by the test instrumentation (power meter, spectrum analyzer, etc.). This arrangement is shown schematically in Figure 1.

Since the test instrumentation can only respond to the signal delivered to it, the value indicated by the test equipment must be corrected to remove the effects of the intervening two-port. This involves finding a two-port gain in the form

$$P_i = \frac{P_{meas}}{G} \qquad (1)$$

where:

P_i is the power incident at the test plane,

P_{meas} is power indicated by the test equipment, and

G is the gain of the intervening two-port.

Unfortunately, the term gain (G) can be defined in a number of ways, and often no information is provided to indicate which definition is being used. This paper discusses the distinctions among different definitions of gain, "available power gain" in particular, and describes a number of measurement methods.

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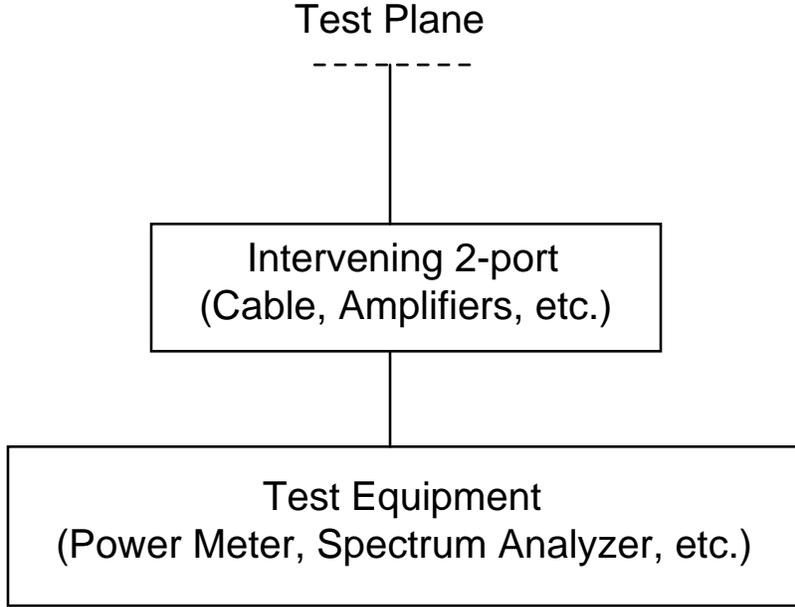
Figure 1. Typical measurement system configuration.

2. WHAT KIND OF GAIN?

When one talks about the gain of a two-port device it is important to specify what one really means. It is not as simple as forming the ratio of the output power versus the input power. As can be seen in Figure 2, there are a number of powers that can be chosen to form this ratio. Here the powers P_{1a} and P_{1d} refer to the powers available and delivered to port one of the two-port, whereas powers P_{2a} and P_{2d} describe the powers available and delivered from port two of the same device. This means that one could offer the following definitions for the gain of the two-port:

$$G_1 = \frac{P_{2a}}{P_{1a}}, \tag{2}$$

$$G_2 = \frac{P_{2a}}{P_{1d}}, \tag{3}$$

$$G_3 = \frac{P_{2d}}{P_{1a}}, \tag{4}$$

$$G_4 = \frac{P_{2d}}{P_{1d}}. \tag{5}$$

Of these gain definitions, equation 2 is known as the "available gain" (G_a) of the two-port, equation 4 is either the "signal gain" (G_s) or "transducer gain" (G_t), and equation 5 is commonly known as simply "power gain" (G) [1]. To the best of my knowledge equation 3 is not named.
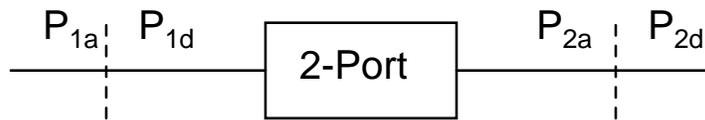

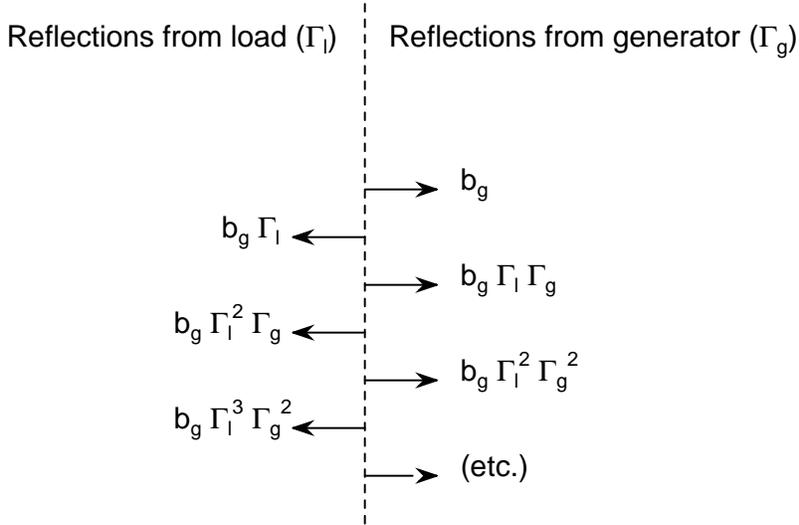Figure 2. Generalized 2-port showing available and delivered powers.

Figure 3. Diagram of multiple reflections at the measurement plane.

Things are further complicated by several other possible gain definitions:

Associated Gain (G_{ass}) The available gain of a device when the source reflection coefficient is the optimum reflection coefficient (Γ_{opt}) [1, pg. 29].

Insertion Gain (G_i) The gain that is measured by inserting the DUT between a generator and a load. The numerator of the ratio is the power delivered to the load while the DUT is inserted, P_d. The denominator, or reference power P_r, is the power delivered to the load while the source is directly connected [1, pg. 31].

Clearly it is important to specify the gain definition one is using.

Choosing the appropriate gain definition requires an understanding of the relationship between available and delivered power. This is a direct result of what can be called the "microwave circuit theory analogue of Thevenin's or Norton's Theorems" [2, pp 18–22] [3, pg 51]

$$b = b_g + a\Gamma_g \tag{6}$$

where:

b is the wave amplitude of the signal emerging from a source,

b_g is the wave amplitude of the generator wave created by the source,

Γ_g is the reflection coefficient seen by energy incident on the source, and

a is the wave amplitude of energy incident on the source coming from the circuit the source is connected to.

By considering the multiple reflections of a transient signal across a reference plane as shown in Figure 3, we can obtain a mathematical description of the mismatch term M. In this case, summing up all of the wave amplitude terms directed to the right we get

$$b = b_g + b_g\Gamma_g\Gamma_l + b_g(\Gamma_g\Gamma_l)^2 + \dots \tag{7}$$

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which can be factored to give us

$$b = b_g[1 + \Gamma_g\Gamma_l + (\Gamma_g\Gamma_l)^2 + \ldots].\tag{8}$$

Recognizing that the factor consisting of an infinite sum of reflection coefficients is a geometric series that converges to $(1 - \Gamma_g\Gamma_l)^{-1}$ as long as $|\Gamma_g\Gamma_l| < 1$ allows us to write this as

$$b = \frac{b_g}{1 - \Gamma_g\Gamma_l}.\tag{9}$$

Note that by definition the power delivered to a passive termination is

$$P_d = |b|^2 - |a|^2,\tag{10}$$

and that

$$a = b\Gamma_l.\tag{11}$$

This means equation 10 can be rewritten in terms of power as

$$P_d = |b|^2[1 - |\Gamma_l|^2].\tag{12}$$

Substituting equation 9 into equation 12 results in the general expression:

$$P_d = \frac{|b_g|^2}{(1 - |\Gamma_g|^2)}\frac{(1 - |\Gamma_g|^2)(1 - |\Gamma_l|^2)}{|1 - \Gamma_g\Gamma_l|^2} = P_a M\tag{13}$$

where the available power P_a is given by

$$P_a = \frac{|b_g|^2}{1 - |\Gamma_g|^2},\tag{14}$$

and the mismatch M is

$$M = \frac{(1 - |\Gamma_g|^2)(1 - |\Gamma_l|^2)}{|1 - \Gamma_g\Gamma_l|^2}.\tag{15}$$

With this in mind, let's assume that we want to measure the power of some arbitrary signal source. Working in terms of available power, the maximum power that this source can supply to a conjugate matched load is P_{source}. Hence the maximum power available to the detector (power meter, spectrum analyzer, etc.) is:

$$P_{out} = G_a P_{source}\tag{16}$$

where G_a is the available gain of the intervening two-port. This is diagrammed in Figure 4.
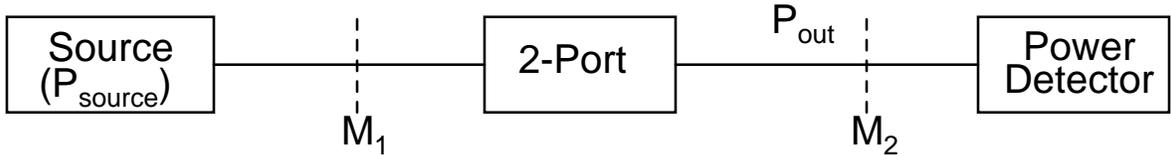

Figure 4. Transfer of power to the detector using "available gain".

It is still necessary to determine the power that will be "indicated" by the detector circuit due to the impedance mismatch M_2 and any calibration factors for the detector. However at this location in the measurement system the engineer usually has control over system design and can design the two-port connecting the signal source to the detector so that either the mismatch term M_2 is known or is constant.

If, on the other hand, we choose to work in terms of the power delivered at the input to this same two-port, we need to know something about the mismatch M_1 between the source and the two-port. This would be described by

$$P_{1d} = M_1 P_{source},\tag{17}$$

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where:

P_{1d} is the power delivered across the measurement plane,

M_1 is the mismatch coefficient at the measurement plane, and

P_{source} is the power available power at the measurement plane.

The power delivered to the detector could then be found using the gain expression given in equation 3. The power transfer equation in terms of delivered power would then be

$$P_{out} = M_1 G_2 P_{source}. \tag{18}$$

This requires measuring values for both M_1 and G_2.

3.   MEASUREMENT UNCERTAINTIES

Before going any further, it is worth noting that no measurement is complete unless there is some specification describing how close the measured value comes to reporting the truth. For example, what if the gain of a two-port was measured to be 50 dB one time and 46 dB the next? Given this information it appears that there is a 4 dB difference between these measurements. However, if this is a measurement of the same two-port under the same conditions do these measurements have a discrepancy of 4 dB? Perhaps, but what if the measurement system had an uncertainty of ± 2 dB? In this case, it is entirely possible that the system may be measuring correctly and still show a 4 dB difference between values. What if the signal path had roughly 1 dB of gain or loss? Does a measurement of 1 dB ± 2 dB mean anything at all? We need to know the measurement uncertainty in order to know the effective resolution of our measurements, and to ensure that our data and measurement process can actually support the conclusions we draw from them.

Unfortunately experiment design and uncertainty analysis is too broad a topic to cover here. There are, however, a number of sources for more detailed information. See [4] and [5]; the analyses that follow are based on these two references. The basic idea is to establish some estimate of the measurement uncertainty inherent in the measurement system (systematic uncertainty) and to combine this with any uncertainties caused by statistical or random fluctuations. However, the use of the term "systematic uncertainty" has recently been dropped in favor of the following definitions [4]:

Type A Uncertainties arrived at by statistical methods,

Type B Uncertainties which are evaluated by other means.

This paper uses the modern Type A and Type B uncertainty definitions.

Finally, it is worth pointing out the difference between uncertainties expressed in linear (fractional or percent) and logarithmic (decibel) terms. It is common to express the uncertainty in terms of a plus and minus value ($\pm \Delta x$). This works well when using fractional or percent change. However, since the decibel scale is logarithmic the plus value does not imply the same magnitude of change as the minus value. Indeed, a statement of ± 3 dB indicates a plus 100% and minus 50% change in value. This means that uncertainties expressed in dB need to be converted to linear (fractional or percent) uncertainty before being combined using traditional error propagation equations.

To convert between fractional uncertainty and dB uncertainty consider the dB representation of a given value x with respect to a reference value y

$$dB_1 = 10 \log(\frac{x}{y}), \tag{19}$$

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and its fluctuation

$$dB_2 = 10\log(\frac{x + \Delta x}{y}).$$ (20)

From this the change in dB can be found as

$$dB_2 - dB_1 = \Delta dB = 10\log(\frac{x + \Delta x}{y}) - 10\log(\frac{x}{y}) = 10\log(\frac{x + \Delta x}{x})$$ (21)

or

$$\frac{\Delta x}{x} = 10^{(\frac{\Delta dB}{10})} - 1.$$ (22)

Since the magnitude of the change depends on whether one is using plus or minus in the dB scale, the convention taken through the rest of this paper is to assume the worst case change.

4. MEASUREMENT METHODS

4.1 Measuring G_a Using a Signal Generator

One common method for measuring gain is to use a calibrated signal generator to supply a source of known available power at the input to the network. With this known input one then observes the power indicated by the detector and forms the ratio. The basic test setup is described in Figure 5.

Figure 5. Test setup for using a signal generator to measure gain.

However, is it sufficient to supply only a single known power, read the detector response and form the power ratio (single-point calibration), or is it better to use two (or more) known levels from which the system gain can be determined (two-point calibration)? Two possible gain curves are shown in Figure 6. Since the use of a single known input and measured output power only identifies a single point on the curve it is clear that this can only work for systems that have no zero point offset. In many real world measurement systems the presence of detector offset, DC bias effects, or appreciable system noise can violate this condition. In these cases, one must resort to the two-point method so that the zero point offset can be taken into account.

The following describes some of the key considerations between the single-point and two-point methods:

- Single-Point Method:

  1. This is an absolute value measurement.
  2. Assumes strict system linearity.
  3. No zero point offset allowed.
  4. Simple test.

- Two-Point Method:

  1. Difference rather than absolute value measurement.
  2. Identifies zero point offset (can be used to linearize the system).
  3. More complicated since two points must be measured.

This is a good example of how the same hardware can be combined in different ways to produce very different measurement methods.
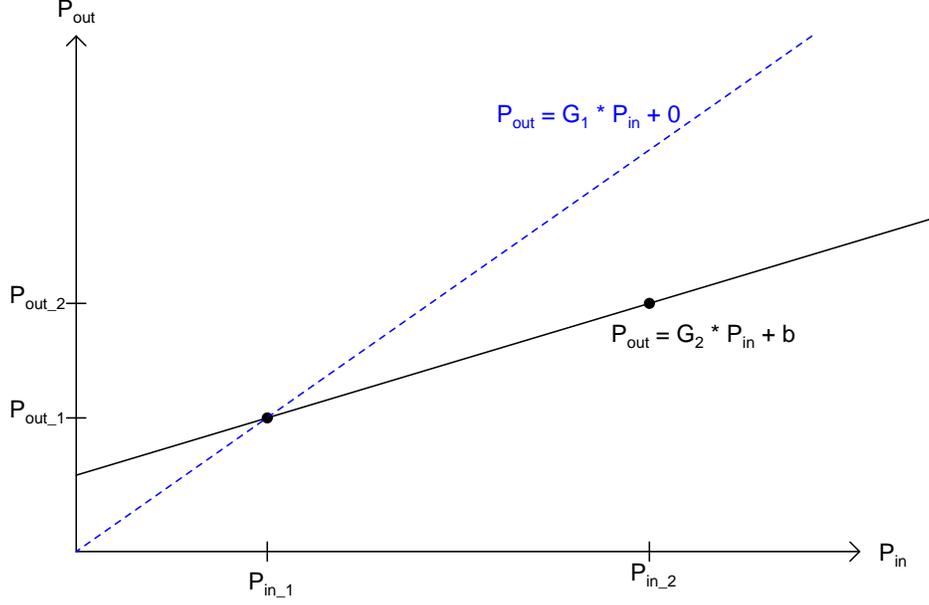
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Figure 6. Two possible gain curves.

4.1.1 Single-Point Calibration Uncertainty

In this case we know the available power supplied at the input to the two-port (P_{a1}), and the detector circuit indicates the power delivered to it from the two-port (P_{2d}). Since the available gain is the ratio of the power available at the output of the two-port to the power available at the input of the device, we make use of the relationship

$$P_{2d} = P_{2a} M_2 \tag{23}$$

where:

P_{2d} is the power delivered to the power detector,

P_{2a} is the power available at the input to the detector, and

M_2 is the mismatch between the two-port and the detector.

Solving for P_{2a} and substituting this into equation 2 gives

$$G_a = \frac{P_{2a}}{P_{1a}} = \frac{P_{2d}}{M_2 * P_{1a}}. \tag{24}$$

As long as we can obtain estimates for the variability of each of the terms in equation 24 (ΔP_{2d}, ΔM_2, and ΔP_{1a}), we can determine the Type B uncertainty by finding the following fractional uncertainties:

$$u_{P_{1a}} = |(\frac{\partial G_a}{\partial P_{1a}})(\frac{1}{G_a})\Delta P_{1a}| = \frac{\Delta P_{1a}}{P_{1a}}, \tag{25}$$

$$u_{P_{2d}} = |(\frac{\partial G_a}{\partial P_{2d}})(\frac{1}{G_a})\Delta P_{2d}| = \frac{\Delta P_{2d}}{P_{2d}}, \tag{26}$$

$$u_{M_2} = |(\frac{\partial G_a}{\partial M_2})(\frac{1}{G_a})\Delta M_2| = \frac{\Delta M_2}{M_2}. \tag{27}$$

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Assuming that these uncertainties are independent we can combine them in quadrature to get a total Type B uncertainty

$$u_B = \sqrt{u_{P_{1a}}^2 + u_{P_{2d}}^2 + u_{M_2}^2}. \tag{28}$$

In order to complete this analysis the Type A uncertainty is commonly found as the standard deviation divided by the mean of a series of repeated gain measurements. Once this value (u_A) is known the expanded measurement uncertainty can be found as

$$U = k\sqrt{u_A^2 + u_B^2} \tag{29}$$

where k is defined by the National Institute of Standards and Technology (NIST) as the "coverage factor" [4]. The expanded uncertainty defines an uncertainty interval about which the measurement result is confidently believed to lie, and the value of the coverage factor is chosen based on the desired level of confidence. Typically, k is in the range of 2 to 3 with $k = 2$ defining an interval with approximately a 95% level of confidence.

4.1.2 Two-Point Calibration Uncertainty

Supplying two known powers (P_{in_1}, P_{in_2}) at the input to the two-port, and measuring the output responses $(P_{a\_out\_1}, P_{a\_out\_2})$ allows us to compute the gain as

$$G_a = \frac{P_{a\_out\_2} - P_{a\_out\_1}}{(P_{in\_2} - P_{in\_1})}. \tag{30}$$

This is simply a more general form of equation 24. One could even approach the uncertainty analysis for this case as the combination of two single point measurements as was done in the prior section; however, of particular interest is that this two-point method gives us the option of analysing the uncertainties as a ratio of differences rather than a ratio of absolute values. This is important since power differences can often be measured more accurately than absolute levels. In fact, one could actually determine the gain without having to know the absolute values of the input powers P_{in_1} and P_{in_2}, providing the difference between the levels is known. However, let's assume for this uncertainty analysis that we are given the absolute power values for P_{in_1} and P_{in_2}. Let's also assume that we are measuring the output power difference $(P_{out\_2} - P_{out\_1})$ which can be written in terms of available powers $(P_{a\_out\_1}, P_{a\_out\_2})$ as:

$$P_{diff} = P_{a\_out\_2} - P_{a\_out\_1} = \frac{P_{out\_2} - P_{out\_1}}{M_2} \tag{31}$$

Assuming that the mismatch term M_2 is constant, we only need to worry about the uncertainty in difference itself (u_{diff}). Proceeding as for the single point case we have:

$$u_{diff} = |(\frac{\partial G_a}{\partial P_{diff}})(\frac{1}{G_a})\Delta P_{diff}| = \frac{\Delta P_{diff}}{P_{diff}}, \tag{32}$$

$$u_{P1a} = |(\frac{\partial G_a}{\partial P_{1a}})(\frac{1}{G_a})\Delta P_{1a}| = \frac{\Delta P_{1a}}{P_{2a} - P_{1a}}, \tag{33}$$

$$u_{P2a} = |(\frac{\partial G_a}{\partial P_{2a}})(\frac{1}{G_a})\Delta P_{2a}| = \frac{\Delta P_{2a}}{P_{2a} - P_{1a}}. \tag{34}$$

From this an estimate of the Type B uncertainty can be found as

$$u_B = \sqrt{u_{diff}^2 + u_{P1a}^2 + u_{P2a}^2}. \tag{35}$$

Again the Type A uncertainty (u_A) is commonly found as the standard deviation divided by the mean of a series of repeated gain measurements, and the expanded measurement uncertainty computed as

$$U = k\sqrt{u_A^2 + u_B^2} \tag{36}$$

where $k = 2$ is the most common coverage factor [4].

Figure 7. Test setup using a noise source.

4.2   Measuring G_a with a Noise Source

For low noise systems with enough sensitivity, the use of a calibrated noise source rather than a signal generator is an option. This can be particularly useful when performing broadband measurements where a single, narrowband frequency calibration using a signal generator may not be practical or desired. In this case, the calibrated signal is created by some device or object emitting energy according to the Nyquist Theorem [2, pg. 203] as if it were a blackbody radiator. This could be a simulated blackbody radiator such as a temperature controlled resistor [6][7], a gas discharge tube, a celestial radio source such as the supernova remnant Cassiopeia A [8], or an avalanche diode. The hardware configuration for this measurement is shown in Figure 7. This is very similar to the setup used for the single frequency case discussed in Section 4.1; however, in this case the available power from the noise source is represented by the expression

$$P_{noise} = kTB \tag{37}$$

where:

P_{noise} is the noise power,

k is Boltzman's constant ($k = 1.38 \times 10^{-23}$ [Joules/Kelvin]),

T is the "effective" blackbody temperature of the noise source in Kelvins, and

B is the measurement bandwidth in Hertz.

Again it is very important to recognize that the noise power described by Equation 37 is a true "available" rather than delivered power [1, pg. 4][2, pg. 203].

An added complication for analysis of the circuit in Figure 7 is that noise sources internal to the two-port network need to be included. Writing this in terms of available gain and power we have

$$P_{2a} = G_a P_{1a} + N_a \tag{38}$$

where:

P_{2a} is the available noise power at port two,

G_a is the available gain of the two-port,

P_{1a} is the available power at port 1, and

N_a is the noise added by the two-port network.

By plotting this function in Figure 8, a few features of the model can be illustrated. First of all, if the two-port could be connected to a blackbody noise source at absolute zero we would expect the internal noise of the two-port to result in an available noise power of N_a at port two of the device.

Finally, note that the measurement of G_a using a hot and cold noise source is really nothing more than using two defined points in order to compute the slope of the equation for the line.
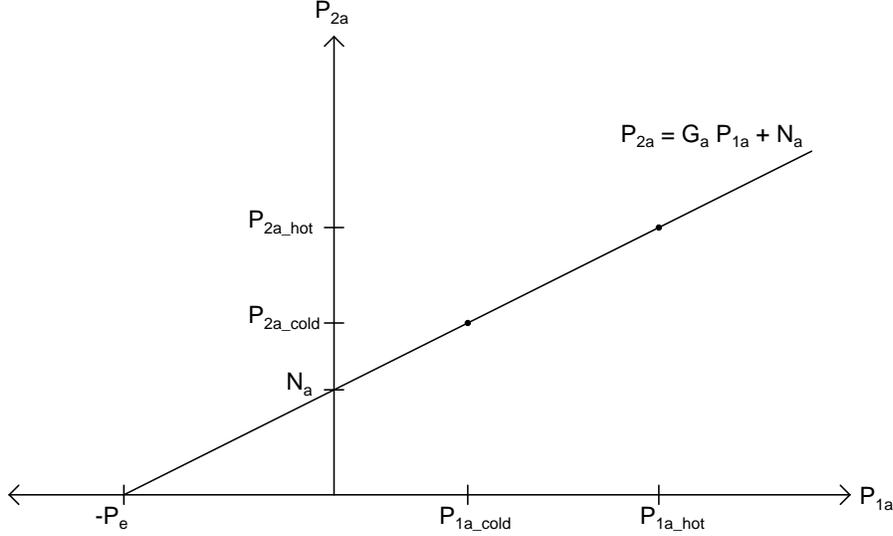
Figure 8. Graphical description of the noise model.

So for two noise sources of known values $P_{1a_{hot}}$ and $P_{1a_{cold}}$ we have

$$P_{2a_{hot}} = P_{1a_{hot}}G_a + N_a \tag{39}$$

and

$$P_{2a_{cold}} = P_{1a_{cold}}G_a + N_a \tag{40}$$

which can be solved for G_a to obtain

$$G_a = \frac{P_{2a_{hot}} - P_{2a_{cold}}}{P_{1a_{hot}} - P_{1a_{cold}}}. \tag{41}$$

Quite often the calibration of the noise source is expressed as an "equivalent" blackbody temperature. This is not usually the actual physical temperature of the device, but rather the temperature a true blackbody radiator would need to have the same power output as the noise source as described by the Nyquist relation. For devices such as noise diodes that can easily be turned on and off it is also common to use the diode "on" temperature to represent $P_{1a_{hot}}$ and the "off" temperature to represent $P_{1a_{cold}}$. If this is done, the "off" temperature usually assumes that of the connected two-port and detector. This has the advantage of establishing the condition where there is no net power flow from the noise source to the connecting two-port as long as these are both at the same temperature. In this case, the unbiased noise source is simply treated as a passive resistor (blackbody) and accepted practice is to assume that the blackbody temperature of this device is then approximately 290 Kelvin. This temperature is commonly represented using the notation T_o. Rewriting equation 41 in terms of the noise source temperature T and T_o results in the following expression.

$$G_a = \frac{P_{2a_{hot}} - P_{2a_{cold}}}{kB(T - T_o)}. \tag{42}$$

Noting that the detector in Figure 7 can only respond to delivered power (P_{2d}), and that there may be a detector calibration factor to consider, the "indicated" power can be described by the expression

$$P_{indicated} = \eta P_{2d} = \eta M_2 P_{2a} \tag{43}$$

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where:

$P_{indicated}$ is the power indicated by the detector,

η is the calibration factor for the detector, and

M_2 is the mismatch between the detector and the two-port.

Letting P_{hot} and P_{cold} represent the powers indicated by the detector with hot and cold noise sources connected to the circuit, allows us to rewrite equation 42 as

$$G_a = \frac{\frac{P_{hot}}{\eta M_{2_{hot}}} - \frac{P_{cold}}{\eta M_{2_{cold}}}}{kB(T - T_o)}. \tag{44}$$

Here it is important to consider the design and operation of the measurement system. For example, the powers P_{hot} and P_{cold} could be measured independently, and assuming that values could be determined for the detector calibration factor η and the mismatches $M_{2_{hot}}$ and $M_{2_{cold}}$, using equation 44 to determine the available gain is straightforward. However, the need to determine the quantities $M_{2_{hot}}$, $M_{2_{cold}}$, and η combined with the implication that the system can make accurate power measurements relative to true power is somewhat problematic. In most cases, the preferred approach would be to design the system to measure the power difference $(P_{hot} - P_{cold})$. It is important however, to note that

$$P_{hot} - P_{cold} = \eta(M_{2_{hot}} P_{2a_{hot}} - M_{2_{cold}} P_{2a_{cold}}), \tag{45}$$

and this is proportional to the numerator in equation 44 only if

$$M_{2_{hot}} = M_{2_{cold}}. \tag{46}$$

4.2.1 Uncertainties when Measuring G_a with a Noise Source

To compute the uncertainties in the gain measurement described by equation 44 we need to find:

u_{diff} the uncertainty in the difference $\frac{P_{hot}}{\eta M_{2_{hot}}} - \frac{P_{cold}}{\eta M_{2_{cold}}}$,

$u_{bandwidth}$ the uncertainty in the measurement bandwidth B,

u_T the uncertainty in the noise source temperature T, and

u_{T_o} the uncertainty in the system temperature T_o.

Since Boltzman's constant k is a universally accepted physical quantity it can be ignored as an uncertainty source for this analysis.

Most test equipment, such as a spectrum analyzer, is designed primarily to measure power differences rather than absolute powers. So it will be assumed that the system is carefully designed to meet the mismatch requirements of equation 46. Furthermore, the following analysis assumes that the measurement detector is well behaved so that not only is $M_{2_{hot}} = M_{2_{cold}}$, these mismatch terms and the calibration factor η are constant. This simplifies the uncertainty analysis since it means the only variability comes from the difference term, $P_{hot} - P_{cold}$. This results in

$$u_{diff} = |(\frac{\partial G_a}{\partial (P_{hot} - P_{cold})})(\frac{1}{G_a})\Delta(P_{hot} - P_{cold})| = \frac{\Delta(P_{hot} - P_{cold})}{(P_{hot} - P_{cold})}. \tag{47}$$

Expressions for $u_{bandwidth}$ and u_{T_o} are

$$u_{bandwdith} = |(\frac{\partial G_a}{\partial B})(\frac{1}{G_a})\Delta B| = \frac{\Delta B}{B} \tag{48}$$

and

$$u_{T_o} = |(\frac{\partial G_a}{\partial T_o})(\frac{1}{G_a})\Delta T_o| = \frac{\Delta T_o}{T - T_o}. \tag{49}$$

However the noise source term is a bit more complicated. Assuming the calibration is done using a noise diode the u_T term is the result of a number of separate uncertainty mechanisms that must be accounted for. This results in the expansion of the single u_T term into:

$u_{T_{cal}}$ the noise source calibration uncertainty,

$u_{T_{amb}}$ ambient temperature effects on the noise source output,

u_{T_V} the noise source power supply stability,

$u_{T_{interp}}$ uncertainty caused by interpolating between calibration points.

These can be written using the increments of the variable T as:

$$u_{T_{cal}} = |(\frac{\partial G_a}{\partial T_{cal}})(\frac{1}{G_a})\Delta T_{cal}| = \frac{\Delta T_{cal}}{T - T_o}, \tag{50}$$

$$u_{T_{amb}} = |(\frac{\partial G_a}{\partial T_{amb}})(\frac{1}{G_a})\Delta T_{amb}| = \frac{\Delta T_{amb}}{T - T_o}, \tag{51}$$

$$u_{T_V} = |(\frac{\partial G_a}{\partial T_V})(\frac{1}{G_a})\Delta T_V| = \frac{\Delta T_V}{T - T_o}, \tag{52}$$

$$u_{T_{interp}} = |(\frac{\partial G_a}{\partial T_{interp}})(\frac{1}{G_a})\Delta T_{interp}| = \frac{\Delta T_{interp}}{T - T_o}. \tag{53}$$

where:

ΔT_{cal} is the variation in the noise source temperature due to the calibration uncertainty,

ΔT_{amb} is the variation in the noise source temperature caused by ambient temperature fluctuation,

ΔT_V is the variation in the noise source temperature caused by power supply variations, and

ΔT_{interp} describes how much variation can exist in the noise source output value (T) due to interpolating between calibration points.

Assuming that these uncertainties are independent we can combine them in quadrature to obtain an estimate of the total Type B uncertainty

$$u_B = \sqrt{u_{diff}^2 + u_{bandwidth}^2 + u_{T_o}^2 + u_{T_{cal}}^2 + u_{T_{amb}}^2 + u_{T_V}^2 + u_{T_{interp}}^2}. \tag{54}$$

The Type A uncertainty would commonly be found as the standard deviation of a series of repeated gain measurements. Once values have been obtained for both the Type A (u_A) and Type B (u_B) uncertainties the expanded uncertainty of the measurement can be computed as

$$U = k\sqrt{u_A^2 + u_B^2} \tag{55}$$

where the coverage factor $k = 2$ representing a 95% confidence limit is commonly used [4].

4.3   Using the Network Analyzer to Determine G_a

Laboratories equipped with vector network analyzers have the option of computing the available gain of a two-port from measurements of the network's scattering parameters (S_{11}, S_{12}, S_{21}, S_{22}) and reflection coefficient of the signal source (Γ_s). In this case, the power available from the signal source at the input to the two-port is described by equation 14, and the power at the output of the intervening two-port can be described by either [1, pg. 29]

$$P_{2a} = \frac{|b_g|^2|S_{21}|^2(1-|\Gamma_2|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_2^* S_{22}) - \Gamma_s \Gamma_2^* S_{12} S_{21}|^2} \tag{56}$$

or

$$P_{2a} = \frac{|b_g|^2|S_{21}|^2}{|1-\Gamma_s S_{11}|^2(1-|\Gamma_2|^2)} \tag{57}$$

where

$$\Gamma_2 = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}. \tag{58}$$

Since the available gain is given by the ratio of the available output power to the input power as described in equation 2, we can compute the gain using either [1, pg. 29]

$$G_a = |S_{21}|^2 \frac{(1-|\Gamma_s|^2)(1-|\Gamma_2|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_2^* S_{22}) - \Gamma_s \Gamma_2^* S_{12} S_{21}|^2} \tag{59}$$

or

$$G_a = |S_{21}|^2 \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2(1-|\Gamma_2|^2)}. \tag{60}$$

Here it can be readily seen how the available power gain depends only on the source impedance and network parameters and is independent of the load connected at the output of the two-port.

4.3.1   Uncertainties in Measuring G_a with a Network Analyzer

In this case we are faced with five variables (Γ_s, S_{11}, S_{12}, S_{21}, S_{22}) that are complex numbers. The S-parameters are also correlated rather than independent variables, which complicates matters even more. In a case like this, it is usually simplest to write a computer program to solve the gain equation and vary the input variables to empirically determine the gain uncertainty. The difficult part to this approach is deciding how to vary the input variables. One option is to estimate the uncertainties with which the network analyzer can determine the real and imaginary components of S-parameters and reflection coefficients. This information could be gleaned either from the specifications given by the manufacturer of the network analyzer,

from a statistical analysis of measuring a number of devices with known network parameters using the test equipment at hand, or from participation in a measurement assurance or comparison program of some kind.

As far as writing the computer program goes, consider that each complex variable consists of a real and imaginary component $(a + jb)$ for which there are two possible uncertainty offsets. Using our estimate Δx for the determination of each real and imaginary component results in the values:

$$a^+ = a + \Delta x, \tag{61}$$

$$a^- = a - \Delta x, \tag{62}$$

$$b^+ = b + \Delta x, \tag{63}$$

$$b^- = b - \Delta x. \tag{64}$$

which means that for each complex variable in our gain equation there are four possible perturbations:

$$a^+ + jb^+, \tag{65}$$

$$a^+ + jb^-, \tag{66}$$

$$a^- + jb^+, \tag{67}$$

$$a^- + jb^-, \tag{68}$$

and that for our gain equations 59 and 60, which depend on five complex variables, there are $4^5 = 1,024$ possible combinations that need to be computed in addition to the "measured" gain value. Finding the maximum difference between these "fluctuations" and the measured value determines the Type B uncertainty (u_B) in the measurement.

As usual, the Type A uncertainty (u_A) would commonly be found as the standard deviation of a series of repeated gain measurements. The expanded uncertainty would then be computed as:

$$U = k\sqrt{u_A^2 + u_B^2} \tag{69}$$

where the coverage factor $k = 2$ would commonly be used to indicate a 95% confidence interval [4].

5. SUMMARY

It is clear that when building a measurement system it is important to account for the gain (or loss) in the RF measurement path, that there are a number of ways of expressing gain, and that the definition used must be consistent with the manner in which the gain term will ultimately be used. Also, it is important to note that how the measurement is performed has an effect on the results. A comparison of some of the basic features of the gain calibration methods discussed can be found in Table 1.

It should also be clear that the uncertainties attainable by any given technique depend not only on the equipment at hand, but also on how it is used. It should be noted that the uncertainty analyses presented here are minimal. They are intended only as a baseline from which a more complete analysis for a specific experiment can be derived.

Table 1. Comparison of Measurement Methods

Measurement Method	Advantages	Disadvantages
Signal Generator: single-point method	• Simple test to perform, minimum time and equipment. • Narrowband calibration. • Suitable for low gain systems.	• Assumes strict system linearity. • No zero point offset allowed. • Requires absolute power measurements which burdens the detector system.
Signal Generator: two-point method	• Is a difference measurement. This puts less demand on the detector circuit. • Zero point offset is allowed. • Narrowband calibration. • Suitable for low gain systems.	• Is a more complicated test.
Noise Source	• Wide bandwidth characterization. • Noise sources tend to be small and easily integrated into a test system.	• Only suitable for high gain, low noise systems. • Not suitable if a narrowband calibration is needed.
Network Analyzer: Gain computation from S-parameters	• Gain is relatively simple to compute from S-parameters. • Results in a complete mathematical model of signal propagation through the two-port.	• Requires the use of complex and expensive equipment. • Requires measurement of five complex variables.

6. REFERENCES

[1] "Fundamentals of RF and microwave noise figure measurements," Agilent Application Note 57-1, July 1983.

[2] G. F. Engen, *Microwave Circuit Theory and Foundations of Microwave Metrology*, IEEE Electrical Measurement Series 9, London, Peter Peregrinus Ltd., 1992.

[3] D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*, International Series of Monographs in Electromagnetic Waves, London, Pergamon Press, 1967.

[4] B. N. Taylor and C. E. Kuyatt, "Guidelines for evaluating and expressing the uncertainty of NIST measurement results," NIST Technical Note 1297, National Institute of Standards and Technology, 1994.

[5] J. R. Taylor, *An Introduction to Error Analysis - The Study of Uncertainties in Physical Measurements*, Mill Valley, CA, University Science Books, 1982.

[6] W. C. Daywitt, "Design and error analysis for the WR10 thermal noise standard," NBS Technical Note 1071, National Bureau of Standards, 1983.

[7] W. C. Daywitt, "A coaxial noise source for the 1 GHz to 12.4 GHz frequency range," NBS Technical Note 11074, National Bureau of Standards, 1984.

[8] J. D. Kraus, *Radio Astronomy 2nd Edition*. Powell, Ohio, Cygnus-Quasar Books, 1986.