

Distinction Between Radar Declaration and Pulse Burst Detection in 3.5 GHz Spectrum Sharing Systems

Frank H. Sanders



technical memorandum

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ABBREVIATIONS AND SYMBOLS

CBSD	Citizens Broadband Radio Service Device
DPA	dynamic protection area
ESC	environmental sensing capability
EMC	electromagnetic compatibility
FCC	Federal Communications Commission
ITS	Institute for Telecommunication Sciences
LTE	Long Term Evolution
N	ratio of radar declaration interval to radar burst interval: $N = (T_{declare}/t_{burst})$
N_{trunc}	truncated integer value of N : $N_{trunc} = (\text{INT}(N))$
n_{expect}	expected number of radar bursts that will be received within the bound of N
n_{min}	minimum number of detected radar pulse bursts required for radar declaration
NTIA	National Telecommunications and Information Administration
OSM	Office of Spectrum Management
PN	Public Notice
PW	pulse width
PRI	pulse repetition interval
R&O	Report and Order
RF	radio frequency
SAS	spectrum access system
SD	standard deviation
SE	standard error
t_{burst}	expected (or average) interval between successive radar bursts
$T_{declare}$	allowed declaration interval for radar signals
U-NII	unlicensed national information infrastructure

EXECUTIVE SUMMARY

In April 2015, the Federal Communications Commission (FCC) issued a Report and Order (R&O) in FCC Docket 12-354 [1] regarding future spectrum sharing between radar and non-radar systems in the band 3550–3650 MHz (called here the 3.5 GHz band); an FCC Public Notice (PN) [2] followed. The R&O and PN call for the establishment of an environmental sensing capability (ESC)¹ network of monitors working in concert with a Spectrum Access System (SAS) to protect offshore radar receivers from interference from the on-shore 3.5 GHz band terrestrial Citizens Broadband Radio Service Device (CBSD) communications network. There are currently two types of CBSDs: Category A and Category B. Category A CBSDs are lower in power, and limited to indoor or low-height outdoor operations. Category B CBSDs are higher in power, and limited to outdoor operation. Category B CBSDs will only be authorized² for use after an ESC is approved and commercially deployed.

The radar receivers are supposed to be protected from interference by associated SASs that switch co-channel and adjacent-channel 3.5 GHz CBSD devices to other frequencies or even other bands when their aggregate emissions exceed the radar’s interference protection criteria. Such zones are identified for example in [3]. Each ESC receiver indicates to its SAS whether radars are present or not. A network of ESC devices is deployed to ensure the radar operations are protected in littoral areas. (Inland ESCs may be deployed to protect sites not near the coasts where radars may operate at test and training ranges.)

A simplified proposed approach for ESC monitors and an associated SAS network controller to trigger actions for protection of offshore radar receivers is provided in [4]. In that report, it was taken as a given that ESCs and SASs will employ what have been vernacularly called detect-and-avoid schemes for discovery of local radar signals, with that discovery leading to subsequent SAS controller actions to forestall CBSD interference to the radar(s). But the term “detect” in this context is an oversimplification. In reality, there must be two technical stages in the discovery of radar signals by ESC-SAS combinations: first, a radar pulse burst detection stage and then a second stage in which detection of one or more radar pulse bursts causes an ESC-SAS to declare that a radar is present in a specified dynamic protection area (DPA) on some frequency or frequencies. This Technical Memorandum (TM) draws the necessary distinction between the pulse burst detection stage and the declaration stage for radar signals. It explains this distinction mathematically and presents curves that relate the probability of pulse burst detection to the probability of declaring a radar to be present. And it explains the implications of this distinction for the testing and certification of ESCs regarding the length of laboratory time that would be required to verify that ESCs meet FCC requirements.

For ESC test-and-certification, the result of the distinction between detection and declaration is that probabilities of declaration will *not* be (indeed *cannot* be, as it turns out) tested directly, even though the probability of radar declaration within a specified time interval will be a requirement for ESC-SAS performance. Instead, only probabilities of pulse burst detection will be (or can be)

¹ In the R&O definition, the ESC can be an entire *system* typically consisting of multiple sensor nodes that may have their own dedicated infrastructure network, or may use distributed CBSD-based sensing, or a combination of both.

² See 47 C.F.R. Section 96.15 and Section 96.67.

measured directly during ESC certification testing. The probabilities of declaration will be (and must be) subsequently *computed*. The computations for probabilities of declaration will be based on a combination of three parameters: the measured probabilities of radar burst detection; the engineered beam-scanning, frequency hopping, and chirping behaviors of radars that will share spectrum with CBSDs; and the allowed interval(s) for ESC-SAS combinations to declare the presence of radars in DPAs.

DISTINCTION BETWEEN RADAR DECLARATION AND PULSE BURST DETECTION IN 3.5 GHZ SPECTRUM SHARING SYSTEMS

Frank H. Sanders³

Future 3.5 GHz Citizens Broadband Radio Service Device (CBSD) transmitters in the U.S. will share spectrum with incumbent Federal radar systems. To avoid interfering with radar receivers in exclusion zones, CBSDs will be deployed in conjunction with Environmental Sensing Capability (ESC) sensors and associated Spectrum Access Systems (SASs). The ESCs and SASs will employ detect-and-avoid schemes for discovery of local radar signals leading to interference mitigation. But the term “detect” in this context is an oversimplification. In reality, there must be two stages in the discovery of radar signals by ESC-SAS combinations: first, a pulse burst detection stage based on a single burst and then a second stage in which detection of one or more radar pulse bursts causes an ESC-SAS to declare that a radar is present in a protection area on some frequency or frequencies. This Technical Memorandum draws the necessary distinction between the ESC pulse burst detection stage based on a single burst and the ESC declaration stage which may be based on multiple burst detections. It explains this distinction mathematically and relates probabilities of pulse burst detection to probabilities of radar declaration. It further describes the implications of this distinction for the testing and certification of ESCs with regard to the time required to test and certify ESC performance.

Keywords: Citizens Broadband Radio Service Devices (CBSDs); dynamic protection area (DPA); spectrum sharing; environmental sensing capability (ESC); interference monitoring; probability of detection; probability of declaration; radar; radar pulse burst; spectrum access system (SAS); spectrum sharing

1. INTRODUCTION

In April 2015, the Federal Communications Commission (FCC) issued a Report and Order (R&O) in FCC Docket 12-354 [1], regarding future spectrum sharing between radars and non-radar systems, called Citizens Broadband Radio Service Devices (CBSDs), in the band 3550-3650 MHz (called here the 3.5 GHz band); an FCC Public Notice (PN) [2] followed. The R&O and PN call for the establishment of an environmental sensing capability (ESC) network of monitors⁴ working in concert with a Spectrum Access System (SAS) to protect offshore radar

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⁴ As noted in the Executive Summary, the R&O defines “ESC” as an entire *system* typically consisting of multiple sensor nodes that may have their own dedicated infrastructure network, or may use distributed CBSD-based sensing, or a combination of both.

receivers from interference from the on-shore 3.5 GHz band terrestrial communications network. The radar receivers are to be protected from interference by SAS mitigation actions applied to CBSD transmitters, the SAS mitigation itself being initially triggered by inputs from ESCs. The radar receivers are supposed to be protected from CBSD-generated interference by a SAS that switches co-channel (or nearly co-channel) 3.5 GHz CBSD transmitters to other frequencies or even other bands when their aggregate emissions exceed the radar's interference protection criteria. Examples of such mitigation zones are identified in [3].

A simplified proposed approach for ESC monitors and associated SAS network controllers to trigger actions for protection of radar receivers is provided in [4]. In that report, it was assumed that ESCs and SASs will employ what have been vernacularly called detect-and-avoid schemes for the discovery of local radar signals with that discovery then leading to subsequent SAS controller actions to prevent CBSD interference to the radar(s).

But the term “detect” in this context is an oversimplification. In reality, there must be two technical stages in the discovery of radar signals by ESC-SAS combinations: first, a single-burst radar detection stage and then a second stage in which detection of one (the first burst) or more (additional bursts) radar pulse groups causes an ESC-SAS to declare with some specified amount of confidence that a radar is present within a so-called dynamic protection area (DPA) on some frequency or frequencies. It is the declaration of a radar's presence, and not just the detection of a radar pulse burst, that will then cause an SAS to undertake interference mitigation actions.

This Technical Memorandum (TM) draws the necessary distinction between the ESC-SAS pulse burst detection stage and the subsequent ESC-SAS declaration stage for radar signals. It explains this distinction mathematically and presents curves that relate the probability of pulse burst detection to the probability of declaring a radar to be present. And it explains the implications of this distinction for the testing and certification of ESCs.

For ESC test-and-certification, the result of the distinction between detection and declaration is that probabilities of declaration will *not* be (indeed *cannot* be, as it turns out) tested directly, even though the probability of radar declaration within a specified time interval will be a requirement for ESC-SAS performance. Instead, only probabilities of pulse burst detection will be (or can be) measured directly during ESC certification testing. The probabilities of declaration will be (and must be) subsequently *computed*. The computations for probabilities of declaration will be based on a combination of three parameters: the measured probabilities of radar burst detection; the antenna beam-scanning or rotation, frequency hopping and chirping behaviors of radars that will share spectrum with CBSDs; and the allowed interval(s) for ESC-SAS combinations to declare the presence of radars in DPAs. A limited amount of ESC testing can be conducted at the declaration level to ensure that declarations do reliably occur when the necessary radar burst-detection regime conditions have been met. But such testing will not have enough time available to collect enough trials to directly confirm declaration probability to within one per cent (0.01).

2. RADAR PULSE BURST DETECTION

2.1 Radar Pulse Bursts as Seen by Monitoring System Receivers

Radar transmitters emit multiple high-power pulse sequences into space within highly directed spatial antenna beams. As the pulse sequences are generated, several degrees of electronic freedom are potentially exercised from pulse to pulse or sometimes from one group of pulses to the next. These free parameters include the pulse repetition intervals (PRI); pulse widths (PWs); pulse frequencies (frequency modulation within pulses being called chirping and frequency variation from one pulse to the next being called hopping); and phasing (either discretely modulated phase changes within individual pulses, called “chips” or “phase code chips” or else continuous phase shifts called minimum shift keying (MSK)).

Concurrently with some or all of these pulse parameters being varied within the transmitter’s output sequence, the radar antenna steers its beam through space so as to search for or track targets⁵ within some specified volume per unit time. Beam shapes are often either fan-shaped (technically following a $(1/\text{cosecant-squared})$ functional envelope) or else cone-shaped.⁶ These fan beams or cones may be continuously rotated, as with a mechanically mounted antenna, or else may be shifted continuously or discontinuously through spatial volumes via electronic beam steering using phased-element arrays. Phased array antennas can even produce multiple beams at once, with all of the beams being steered independently of each other.

The result of all of these available degrees of freedom for pulse generation and beam steering is that an ESC will never see a pulse sequence generated by the transmitter as a continuous and well-behaved sequence of pulses. Instead, an ESC will see a convolution of all of the pulse modulation and beam-steering behavior of the radar. This convolution will be the overlap of the radar’s pulse sequence and beam steering with the tuned frequency (or frequencies) and bandwidth (or bandwidths) of the monitoring system at its particular location in space at a particular moment in time. Like any monitoring system, an ESC will only get a glimpse at a time of a radar pulse sequence as it is modulated and steered through space. That glimpse will be limited by the ESC’s tuned receiver frequency, antenna pattern, bandwidth, and physical location.

There is a constraint on radar design that is of significance to radar pulse monitoring and detection. This is the requirement that every radar must illuminate (the term used by radar engineers is “paint”) each and every target with a minimum number of pulses in order to achieve a minimally required probability of target detection in the returning stream of echoes. As shown for example in [5 pg.100], this number is never one or two; it is usually about 20 pulses (with some variation from one radar model to the next) for air search radars and maritime navigation radars, and may be between 40 to 50 for meteorological radars and can be tens of thousands for wind profiler radars. This number strikes a balance between reducing radar beam-scanning speed

⁵ In the context of radar engineering, the term “targets” is not military; the word is used for all objects being scanned by any radar.

⁶ Fan beams are usually narrow in azimuth and wide in elevation; they are used for rapid, two-dimensional air scanning and searching. Cones, also called pencil beams, are used for target tracking and three-dimensional volume scanning.

by dwelling longer on targets versus meeting mission objectives for radar target detection performance. To the extent that an ESC's location is treated the same way by the radar as any other point in space, the ESC may expect to see a set of twenty or more radar pulses as the radar's antenna scans or rotates across its location. The number of pulses will be reduced to the extent that the radar transmitter chirps or hops to frequencies outside the ESC's bandwidth within the beam-scanning dwell. But, most likely, at least several pulses at a time will be seen by any ESC receiver whenever the main beam of the radar antenna paints its location.

Figure 1, a measurement of the antenna beam shape of Shipborne Radar 1 as described in [6], shows a set of incident pulses from the radar as its beam paints a stationary location. This radar uses a waveform with a fixed PRI, PW and frequency. Its antenna beam is narrow in azimuth and fan-shaped in elevation. It is steered via a mechanically rotated antenna in the horizontal plane at the rate of one scan (rotation) every 3.9 seconds. It represents the simplest conceivable design of any radar. Note that this radar puts 19 pulses into the monitoring station's receiver within the 3 dB points of its antenna beamwidth, exactly as called for in the tabulated graphs of [5] for non-complex target-detection conditions.

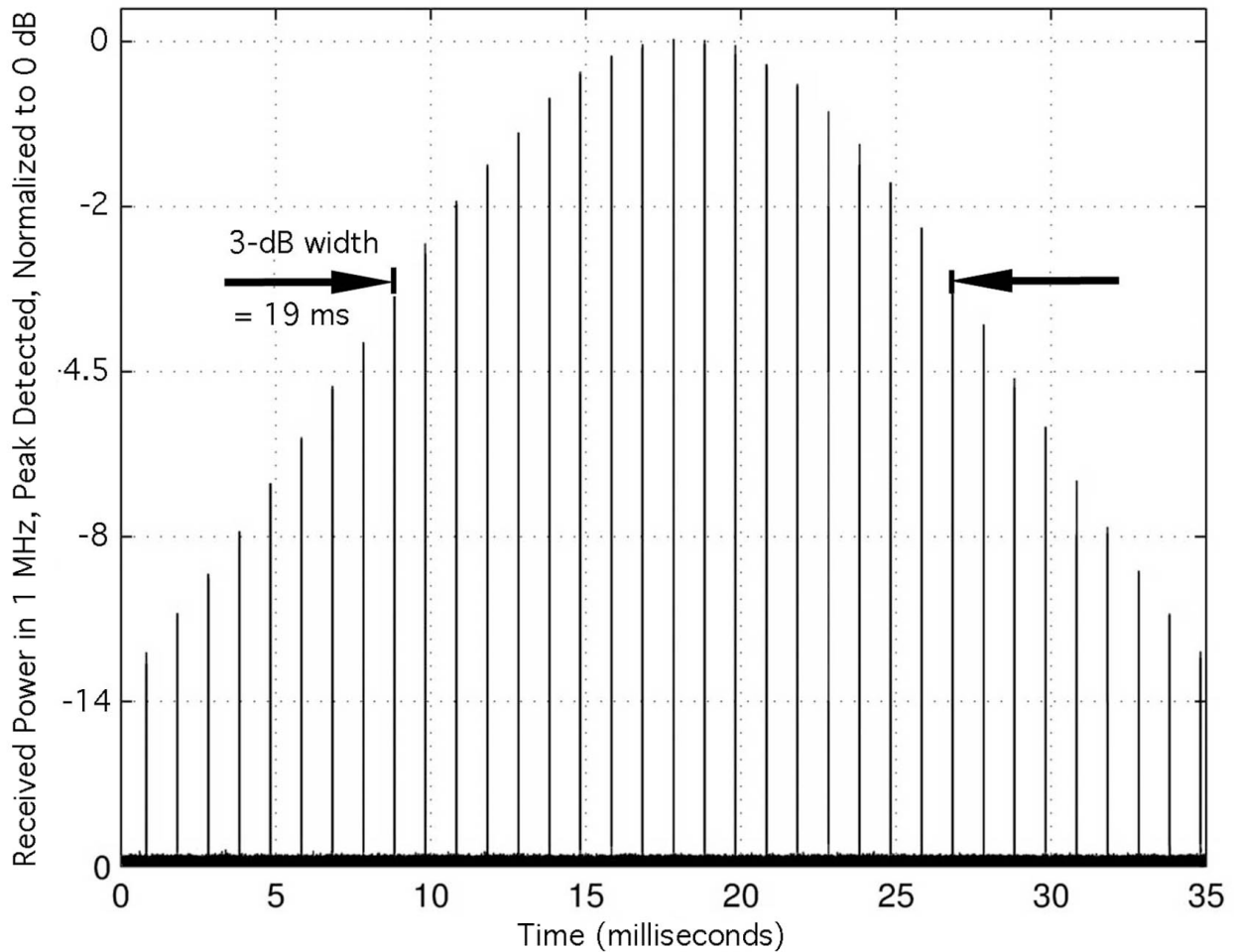


Figure 1. Beam shape of a simple radar showing classical 19 pulses between the 3 dB points.

In contrast to the simplest conceivable radar burst pattern of Figure 1, the radar pulse burst of Figure 2 shows the sequence received by a monitoring station when it is painted by a modern phased array radar. The *inherently non-recurring* pattern in this burst sequence results from the convolutional effect discussed above, mostly due to frequency chirping and hopping as the radar operates. This sort of irregular-looking burst pattern represents the design trend for many modern radars. Note, however, that the apparent irregularity in the burst sequence of Figure 2 is only a function of the monitoring station's lack of knowledge of the radar's operational and technical details. When the radar's design and operational functions are understood, the burst pattern makes perfect sense and can be seen to contain regularity after all. But it must be emphasized that *no two pulse bursts observed from a phased array radar by an ESC or any other monitoring station will ever appear to be identical*—every phased array radar pulse burst will appear to be unique.

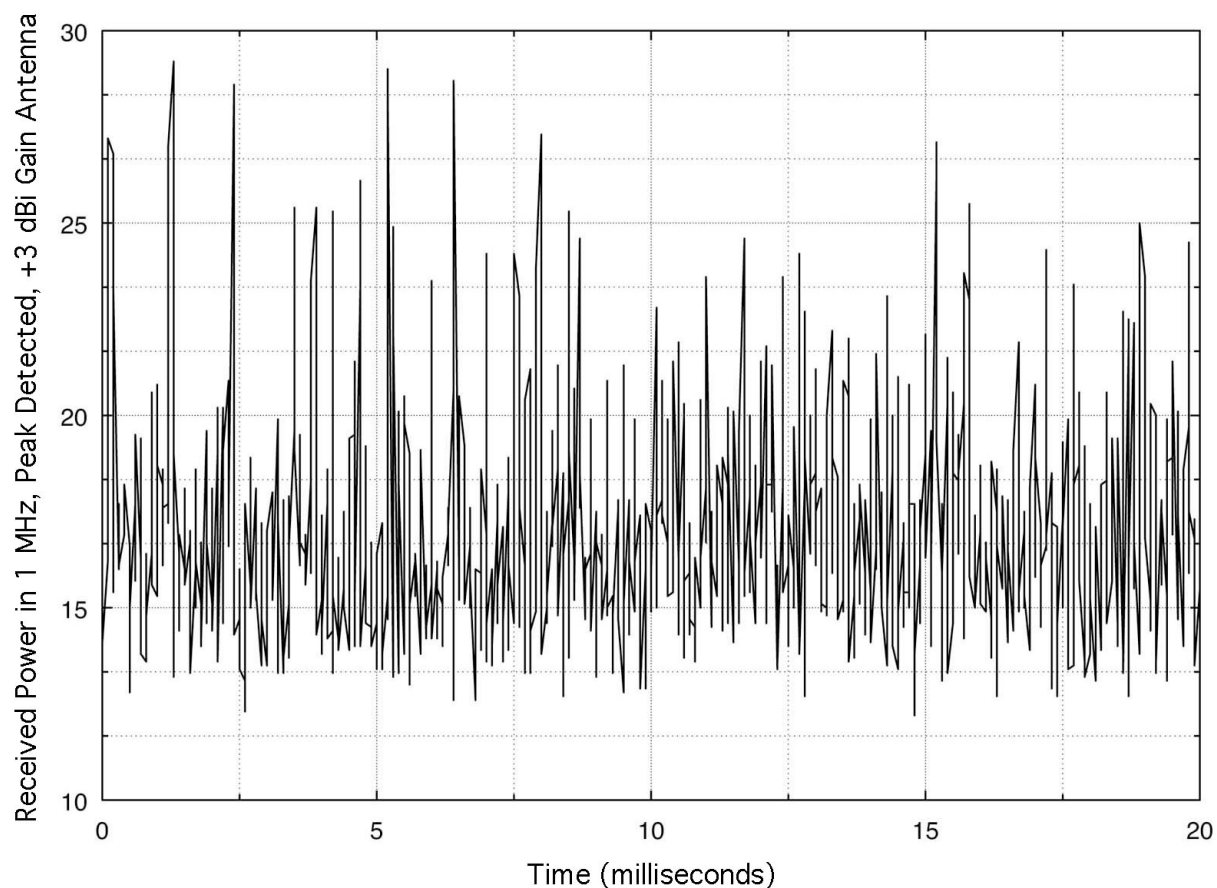


Figure 2. Typical non-replicating pulse sequence received by a monitoring station from a phased array radar that shifts frequencies within and between pulses as it scans its beam through space.

2.2 Pulse Bursts: The Basic Physical Unit of Radar Signal Detection

No matter the exact design of any radar, *some* sort of burst group of radar pulses will always illuminate a monitoring station (i.e., an ESC) whenever a radar paints its location. Given that, it will always be assumed that a *radar pulse burst* will be the *basic physical unit* of the signal

received by any monitoring station or ESC from any radar. All ESC-SAS operations related to sensing radar signals must therefore be based on this physical unit.

This point is important because it means that if pulse burst groups from radar transmitters are not somehow sensed by ESCs, the process of mitigating interference from CBSDs to radar receivers will not progress. While it is entirely conceivable (and possibly desirable) that some sort of ESC-SAS design might require inputs from *multiple* pulse burst groups in order to perform its interference-mitigation mission via CBSD control options, there is no way that an ESC-SAS system can work without *at least one* pulse burst having been sensed. Logically, this means that at a minimum some sort of temporary or place-holder logical flag must be set by an ESC-SAS if it thinks it has just seen (or might have seen) a single radar pulse burst.

The action of “seeing” a single radar pulse burst is called *detection* in this TM. No matter how an ESC-SAS network may address the problem of knowing that a radar is present in a DPA, individual radar pulse bursts *must* be detected with a non-zero probability, P_{detect} .

There are many possible ways of performing pulse burst detection. Approaches may include power threshold exceedance in an ESC receiver coupled with some sort of discrimination gating for PRI, PW, and number of pulses within a burst meeting certain criteria within some expected main-beam painting interval that might be on the order of 20 to 100 milliseconds. This TM does not address those techniques; the author merely notes that some type of time-based sorting and gating for power and pulse characteristics could be a part of the detection process for radar pulse bursts by ESCs and their associated SASs.

3. RADAR DECLARATION

3.1 Moving from Burst Detection to Declaration of a Radar

Detecting a radar pulse burst or multiple bursts is only the beginning of a process that an ESC⁷ will need to implement before its associated SAS begins to undertake interference mitigation by CBSDs. The ESC will need to have some algorithm for ascertaining with some degree of certainty that a radar is truly present in a DPA. This will include the elimination of false positives for burst detections. This process may require enough time for the ESC monitoring system to register multiple radar pulse bursts. According to the Report and Order, an ESC must notify its associated SAS within 60 seconds that radar detection has occurred. The R&O does not however specify if that notification is to be based on a single burst of pulses or multiple bursts.

Ultimately, however, no matter what the detection situation is, if an ESC and its associated SAS are to undertake interference mitigation actions, they will have to reach a logical state in which they have established with some level of confidence that a radar is present within a DPA. In this TM, this state of interference-mitigation action is called *declaration* that a radar is present.

A declaration can conceivably be based on a single-burst detection, but each declaration obviously must be based on *at least one* burst detection. Declaration might as well require multiple detections, but does not have to. Indeed, for many frequency hopping radars it is possible, if not likely, that a single ESC station would only have a single detection on any given frequency channel within the prescribed one minute to declare a radar present. If an ESC provider attempts to correlate detections on all channels, and is constantly staring at (monitoring) all channels with a wideband receiver, then the probability of having more than one detection in a one-minute period is greatly increased. Additionally, in the case of a frequency hopping radar it may be advantageous to use several ESC stations to detect the frequency hopping events from a given radar. The results of this simultaneous collection of frequency hopping burst-pulse events can be assessed to declare the presence of a frequency hopping radar. In any event, declaration is clearly not the same as detection. Because even if a radar declaration is based on a single detection, the ESC must logically progress from the detection event (pulses were registered in a group) to the process starting the implementation of interference mitigation (meaning that a declaration flag or something equivalent has been set positive for a DPA in the ESC).

3.2 $P_{declare}$, Not P_{detect} , as a Primary Requirement for ESC-SAS Certification

The probability of the action of mitigating interference to radars by ESC and associated SAS networks will have a minimum determined value (e.g., 0.90 or 0.95 or 0.99, etc.) determined by regulation and Federal agency requirements. Since the action of mitigation is by definition only initiated when a radar is declared by an ESC, this minimum regulatory probability value will be the probability of declaration of a radar's presence, $P_{declare}$, within a specified time interval,

⁷ The FCC's rules are ambiguous as to exactly what part(s) of radar declaration is/are to be performed by an ESC *versus* which parts, if any, are to be undertaken by an associated SAS. In this discussion the use of the term 'ESC' is taken to encompass the ESC's role in this process along with whatever work the SAS may also do to perform or complete the declaration.

$T_{declare}$, that begins when an operating radar enters a DPA. Conversely, there is *not* any regulatory requirement envisioned for P_{detect} . This dichotomy has interesting consequences for ESC testing and certification, as we shall see.

3.3 Mathematical Relationship Between $P_{declare}$ and P_{detect}

There is a mathematical relationship between $P_{declare}$ and P_{detect} . Three parameters link these two probabilities:

- $T_{declare}$, the total time allowed for a declaration after an operating⁸ radar has entered a DPA, as already noted above;
- t_{burst} , the time interval between radar pulse bursts;
- $N = (T_{declare}/t_{burst})$, a rational number that is the number of radar bursts expected to occur within the interval $T_{declare}$;
- $N_{trunc} =$ the integer truncation (not rounding) of $(T_{declare}/t_{burst})$, used for computational purposes in a combinatorial equation derived below;
- n_{min} , an integer that is the minimum number of radar pulse bursts required by the ESC-SAS to declare a radar within a DPA within interval $T_{declare}$. It should be obvious that this parameter must meet the condition $N \geq n_{min} \geq 1$;
- n_{expect} , an integer that is the *expected* number of radar bursts that will exceed the detection threshold at an ESC location within the interval $T_{declare}$. n_{expect} will be equal to N_{trunc} if propagation conditions are excellent; otherwise, n_{expect} will tend to be less than N_{trunc} under fading propagation conditions such as depicted schematically in Figure 3. The condition $n_{expect} \geq n_{min} \geq 1$ must hold for radar detection to succeed.

⁸ Radar transmitters do not necessarily operate at all times. A radar might enter a DPA when it is not transmitting and then turn on at a later time. The time interval for radar declaration by an ESC begins when a radar meets the twin conditions of being within the DPA and transmitting a signal.

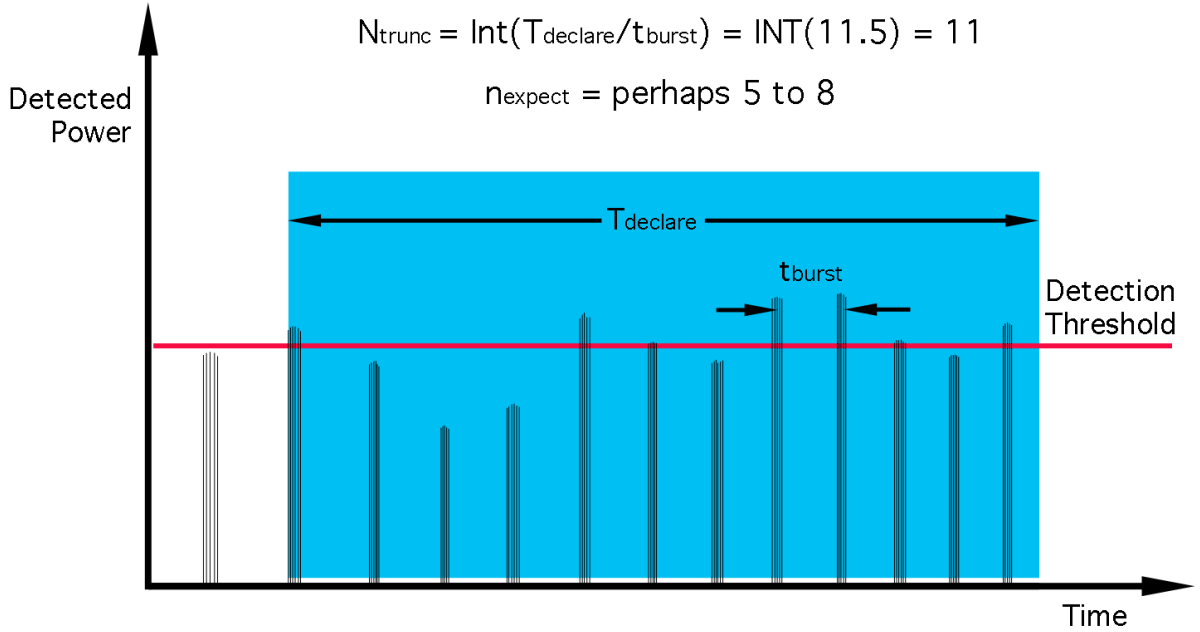


Figure 3. Diagram showing relationships among $T_{declare}$, t_{burst} , N and N_{trunc} for a hypothetical radar with a fixed beam-scanning interval. (For phased array radars, t_{burst} will vary from one event to the next; an average must be used for this parameter.)

The interval t_{burst} is fixed and deterministic for mechanically rotated radar antennas with non-dynamic beam shapes, such as the radar whose emissions are portrayed in Figure 1. For phased array radars, however, t_{burst} will be neither fixed nor deterministic. It will instead be a statistically determined average (e.g., every 12.4 seconds on the average) for such radars.

For purposes of computing $P_{declare}$, the rational number $N = (T_{declare}/t_{burst})$ is truncated (not rounded) to the next-lowest integer, N_{trunc} .

If the probabilities of detecting radar bursts are independent⁹ from one burst to the next, then the probability of declaring a radar within interval $T_{declare}$ is equal to the sum of the probabilities of detecting at least n_{min} bursts within that interval:

⁹ It is conceivable that an ESC might learn sequential radar pulse bursts for some radars, so that the burst-detection probabilities are not independent. But the uniqueness of every phased-array radar pulse burst and the lack of a uniform time interval between phased array pulse bursts (that is, the lack of any cyclostationary characteristics in phased array radar pulse bursts) make such a process unlikely to succeed for detection of the phased array radar signals transmitted by modern radars. Even for regular, mechanical radar beam rotation of older radars, temporal variation in propagation conditions means that there is no assurance that sequential radar scans will be received by an ESC, as drawn schematically in Figure 3. This analysis therefore proceeds with the assumption that no cyclostationary advantage can be gained from successive radar bursts and that detections of individual bursts need to be treated as probabilistically independent events.

$$P_{declare} = P(n_{min}, N_{trunc}) + P(n_{min} + 1, N_{trunc}) + P(n_{min} + 2, N_{trunc}) + \dots + P(n_{expect}, N_{trunc}) \quad (1)$$

where $P(i, N_{trunc})$ is the number of combinations of N_{trunc} things taken i at a time multiplied by the twin probabilities of getting exactly i detections and exactly $(N_{trunc} - i)$ non-detections:

$$P(i, N_{trunc}) = \binom{N_{trunc}}{i} \cdot (P_{detect})^i \cdot (1 - P_{detect})^{N_{trunc}-i} \quad (2)$$

The number of combinations of N_{trunc} things taken i at a time is:

$$\binom{N_{trunc}}{i} = \frac{N_{trunc}!}{i! \cdot (N_{trunc} - i)!} \quad (3)$$

So the complete expression for $P_{declare}$ is:

$$P_{declare} = \sum_{i=n_{min}}^{n_{expect}} \left(\frac{N_{trunc}!}{i! \cdot (N_{trunc} - i)!} \right) \cdot (P_{detect})^i \cdot (1 - P_{detect})^{N_{trunc}-i} \quad (4)$$

In the special case in which $n_{min} = 1$ and $n_{expect} = N_{trunc}$ this expression reduces to:

$$P_{declare} = 1 - (1 - P_{detect})^{N_{trunc}} \quad (5)$$

which is simply unity minus the probability that exactly zero detections occur within the interval N_{trunc} . That is, it is unity minus the probability of getting N_{trunc} non-detections in a row.

4. EXAMPLE $P_{DECLARE}$ CALCULATIONS AND CURVES

4.1 Example Computation of $P_{declare}$

To make the mathematics more accessible, an example calculation of $P_{declare}$ is provided for a simple case. Consider the interval $T_{declare} = 50$ seconds, $t_{burst} = 9$ seconds on average (for some sort of phased array radar in some scan mode), $P_{detect} = 0.60$, the ESC is sufficiently capable that $n_{min} = 1$ and propagation conditions are excellent so that $n_{expect} = N_{trunc}$. The ratio of $T_{declare}$ to t_{burst} is $N = (50/9) = 5.56$ which truncates to $N_{trunc} = 5$. From this starting point the computation of $P_{declare}$ proceeds as shown in Table 1, where $C(N_{trunc}, i)$ is the number of combinations of n_{expect} ($= N_{trunc}$) things taken i at a time.

Table 1. Example calculation of $P_{declare}$.

$P(i, N_{trunc}) = C(N_{trunc}, i) \cdot (P_{detect})^i \cdot (1 - P_{detect})^{N_{trunc} - i}$	Product Expression	Decimal Value
$P(1, 5) = C(5, 1) \cdot (0.6)^1 \cdot (0.4)^4$	$(5) \cdot (0.6) \cdot (0.0256)$	0.07680
$P(2, 5) = C(5, 2) \cdot (0.6)^2 \cdot (0.4)^3$	$(10) \cdot (0.36) \cdot (0.064)$	0.23040
$P(3, 5) = C(5, 3) \cdot (0.6)^3 \cdot (0.4)^2$	$(10) \cdot (0.216) \cdot (0.16)$	0.34560
$P(4, 5) = C(5, 4) \cdot (0.6)^4 \cdot (0.4)^1$	$(5) \cdot (0.1296) \cdot (0.4)$	0.25920
$P(5, 5) = C(5, 5) \cdot (0.6)^5 \cdot (0.4)^0$	$(1) \cdot (0.07776) \cdot (1)$	0.07776
		$\Sigma = P_{declare} = 0.98976$

As a check on the summation result of Table 1, we take advantage of the fact that $n_{min} = 1$ and directly compute the equivalent single expression for a series of five non-detections in a row: $P_{declare} = (1 - (1 - 0.6)^5) = 0.98976$. This confirms the Table 1 result.

4.2 Example $P_{declare}$ Graphs

In Figure 4, $P_{declare}$ is graphed as a function of $N_{trunc} = n_{expect}$ with $n_{min} = 1$ for four different values of P_{detect} . Figure 5 shows the same curves graphed over the restricted range of $P_{detect} = 0.9$ to 1. The curves show that for $P_{detect} = 0.4$, N_{trunc} would need to be 9 or larger to reach a value of $P_{declare} = 0.99$. When $P_{detect} = 0.6$ N_{trunc} would only need to be 5 to achieve that probability of declaration (and as shown in the computation in the preceding section). And when P_{detect} is 0.8 N_{trunc} would only need to be 3 to reach this level of probability for declaration of a radar's presence.

Conversely, the graphs of Figures 4 and 5 can be used to estimate the value of P_{detect} (for $n_{min} = 1$) that would be needed to achieve a required $P_{declare}$ level for a fixed value of $N_{trunc} = n_{expect}$. For example, if N_{trunc} were fixed at 6 and $P_{declare}$ were required to be 0.99 or better, then the level of P_{detect} would need to be 0.55 or better per radar pulse burst.

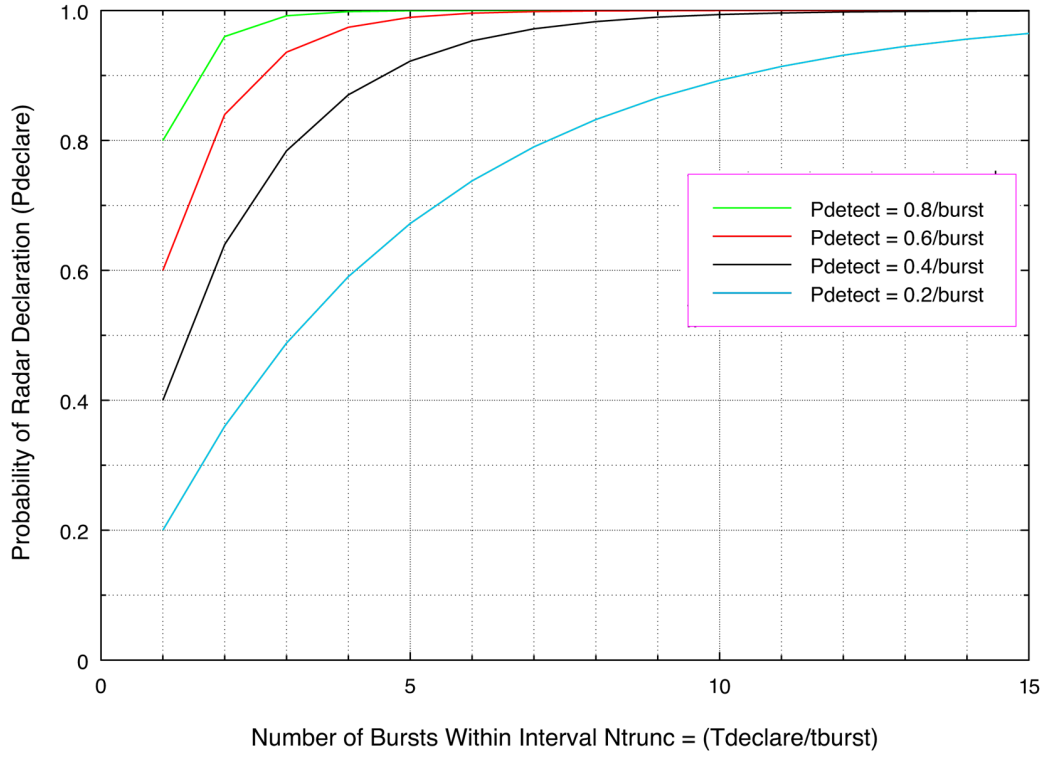


Figure 4. $P_{declare}$ as a function of bursts within N_{trunc} (for $n_{min} = 1$) for four values of P_{detect} .

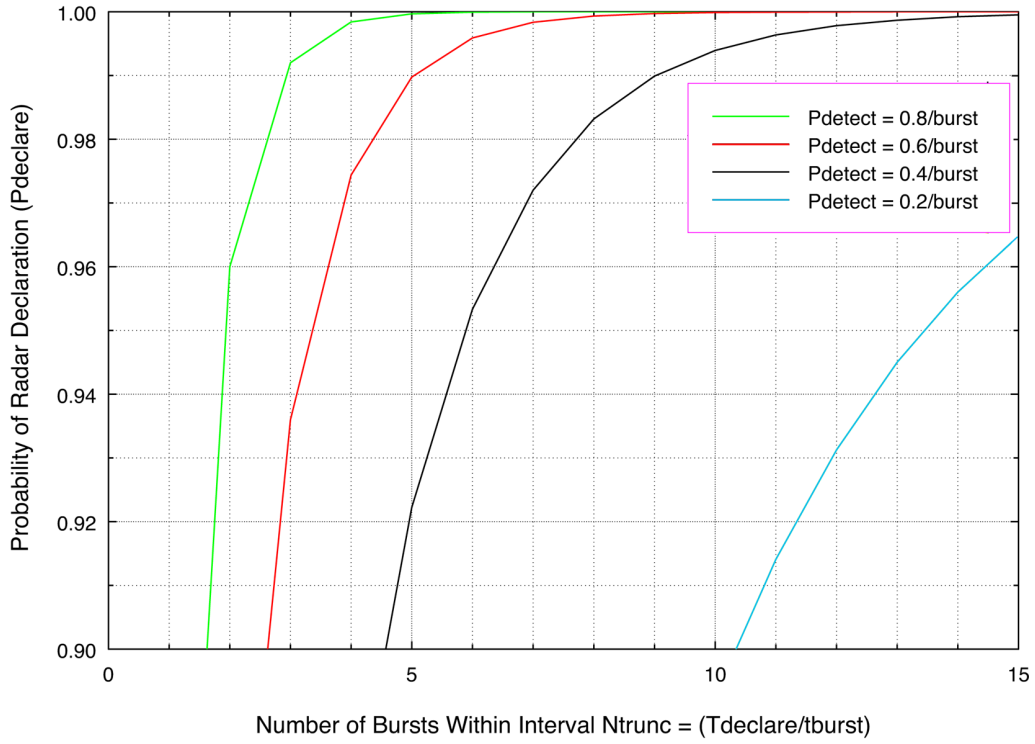


Figure 5. Restricted-range ($P_{declare} = 0.9$ to 1) graph of Figure 4 curves.

5. IMPLICATIONS FOR ESC TESTING AND CERTIFICATION

P_{detect} , not $P_{declare}$, needs to be the ESC performance quantity that is measured during tests and certification. The reasons are:

- P_{detect} is the fundamental unit of radar sensing performance.
- $P_{declare}$ is a secondary function of P_{detect} ; it is derived from P_{detect} .¹⁰
- $P_{declare}$ depends on the ratio of ($T_{declare}/t_{burst}$); its value of N_{trunc} may not necessarily be known when ESC tests are performed because of uncertainty about one or both of the intervals in the ratio.
- Even if the value of N_{trunc} is known when ESC tests are performed, it may change over time due to subsequent changes in either $T_{declare}$ or t_{burst} . But if P_{detect} values are known for a radar, a subsequent change in N_{trunc} can be accommodated by a simple re-calculation of $P_{declare}$ using the new values of $T_{declare}$ and t_{burst} without needing to repeat ESC certification testing.
- The statistical standard error (SE) of $P_{declare}$ for radar waveform trials will vary with the inverse of the square of the SE that is required of the data set, assuming that the probability of detection in each trial is normal (Gaussian) distributed.¹¹ As explained in Appendix A, if $P_{declare}$ were about 0.90, and if $P_{declare}$ needs to be measured to within one per cent (0.01), then about 900 trials, each of duration $T_{declare}$, will be needed *for each radar waveform iteration* to achieve that level of statistical certainty. Two working days—900 minutes—is thus the testing time that would be needed to measure a single radar waveform data point for $P_{declare}$. To measure 100 total data points across all radar bins for an ESC certification data set would require 200 working days or about ten working months. This is probably not a feasible time scale for testing and certification of an ESC.
- Conversely, suppose that P_{detect} (as opposed to $P_{declare}$) is 0.6. Now, it turns out that P_{detect} only needs to be measured to *one place after the decimal* (0.1) to obtain a value of $P_{declare}$ that is good to *two places after the decimal* (1/100), for even a small value of N_{trunc} . This is shown for an example case of $N_{trunc} = 5$ and $n_{min} = 1$ in Table 2. (This result is fundamentally rooted in the behavior of exponents.) Then as described in Appendix A, the required number of n trials for P_{detect} shrinks down to about 24.

¹⁰ $P_{declare}$ will be spot-checked during ESC testing to gain some confidence that it occurs as expected. But as this TM shows, it will not be possible to measure it directly to a confidence level of 0.01.

¹¹ There is no reason to expect that the ESC detection-declaration probabilities will not be normal-distributed; the normal distribution is the best default to use unless other information about the probability distribution comes to light. While the normal distribution assumption breaks down when the true-positive probability is close to unity, we cannot know in advance that actual ESC performance will be so close to unity. Even if ESC performance is close to unity, the binomial analysis of Appendix A still gives a conservative estimate of the number of trials needed to complete ESC testing. Conservatism in resource estimates for testing is necessary when critically important testing timelines and monetary resources are at stake.

- For a P_{detect} instead of a $P_{declare}$ approach to ESC testing and certification, the time per trial needs to be no longer than what is required for the ESC to sense whether a radar pulse burst has occurred. This interval ought to be on the order of a few seconds per trial, at the most.¹² Along with the reduction in n noted above (about 900 for $P_{declare}$ reduced to about 24 for P_{detect}), this reduction in time per trial from $T_{declare}$ to only a few seconds will enormously increase the rate of certification testing. That will, in turn, substantially reduce the cost of testing each ESC.
- If $T_{declare}$ is the time allotted for each trial to try to measure $P_{declare}$ directly, then for simulations of phased array radars the problem of exactly how to structure successive radar bursts within $T_{declare}$, and at what intervals those bursts should be provided, has no clear solution and perhaps no solution at all. This is because real-world phased-array radars will not paint an ESC with bursts at fixed time intervals and furthermore the pulse structure will vary from burst to burst for such radars. No testing protocol is envisioned to try to replicate phased array radar bursts at irregular intervals with varying burst structure from one to the next.
- Some direct observations of $P_{declare}$ can be made in the course of ESC certification testing. But due to the long (e.g., one minute) intervals required for each direct-observation $P_{declare}$ trial, such observations can only confirm that declarations occur with some sort of reasonable reliability when the necessary radar burst-detection regime conditions have been met. There is not enough time available in a series of certification tests to collect enough declaration trials to confirm directly that declarations occur with a likelihood that is known to within one part in one hundred (0.01).

Table 2. Behavior of $P_{declare}$ when P_{detect} is known only to 1/10 ($N_{trunc} = 5$, $n_{min} = 1$).

P_{detect} (0.1 Between Values)	$P_{declare} = 1 - (1 - P_{detect})^5$	Delta Between $P_{declare}$ Values
0.5	0.96875	
		0.02101 ($\approx 1/50$)
0.6	0.98976	
		0.00781 ($\approx 1/125$)
0.7	0.99757	

¹² An interval of a few seconds for each P_{detect} trial is consistent with the author's past experience with similar radar-sensing systems in the 5 GHz unlicensed national information infrastructure (U-NII) bands. The U-NII experience has shown that even low-cost, non-dedicated listening devices can routinely and very reliably produce positive detections of radar bursts within a half a second or so when exposed to radar signals. This kind of U-NII result is achieved with monitoring that is performed in-between the U-NII transmitter's own data packets.

6. SUMMARY AND CONCLUSION

6.1 Summary

Mitigation of interference to radar receivers by ESCs and their associated SASs requires two stages for radar sensing: detection of individual radar pulse bursts followed by declaration that a radar is present. There are probabilities associated with both detection and declaration, P_{detect} and $P_{declare}$. Pulse burst detection is the fundamental unit of this process. But $P_{declare}$ is the metric which will have a regulatory requirement for ESC performance.

P_{detect} and $P_{declare}$ are mathematically related. The relationship is via the parameters of the declaration interval, $T_{declare}$, that is allowed for ESCs to sense a radar within a DPA; the fixed or average intervals between radar pulse bursts, t_{burst} ; and the minimum number of detected radar pulse bursts, n_{min} , that an ESC needs to see within the interval, $T_{declare}$, in order to declare that a radar is within a DPA and that mitigation action needs to commence.

A number of problems make direct measurement of $P_{declare}$ extraordinarily difficult. These include:

- Difficulties with either knowing the ratio $(T_{declare}/t_{burst}) = N$ when ESC certification testing is performed or else with the fact that N may change subsequent to ESC testing having been performed.
- For an interval $T_{declare} = 1$ minute, it will take two working days to obtain a single data point on a single radar waveform if the certainty of $P_{declare}$ needs to be within one per cent. It could take 200 working days to collect 100 data points for a single ESC. There is simply not enough time and funding available to perform such testing.
- Due to the behavior of exponents, if P_{detect} is measured to a certainty of only 1/10, the value of $P_{declare}$ can be computed to within about 1/100.
- Measuring P_{detect} to only a factor of 1/10, with only a few seconds required to determine whether any single radar burst has been detected, will make ESC testing practical in terms of both available time and funding.
- If P_{detect} rather than $P_{declare}$ is measured for each radar waveform during ESC testing, then $P_{declare}$ can always be re-computed at a later date from the existing data if any of the factors $T_{declare}$, t_{burst} , or n_{min} should ever change. The P_{detect} testing approach will obviate the need to re-test ESCs in such an eventuality.
- Testing ESCs for responses to individual radar pulse bursts obviates difficulties with building multiple phased-array pulse bursts that would be needed within each interval $T_{declare}$ for direct $P_{declare}$ ESC certification testing. Those phased-array certification testing difficulties are two-fold: no two phased array pulse bursts as seen by an ESC will ever be identical in the real world, and the intervals between phased array radar bursts arriving at ESC stations will be variable. Doing single-burst-per-trial P_{detect} ESC certification testing eliminates these difficulties.

6.2 Conclusion

ESC certification testing will be most efficiently and effectively performed with a single radar burst per trial, resulting in a set of direct measurements of P_{detect} performance for each ESC for each tested radar waveform. Each P_{detect} trial should require just a few seconds (e.g., 5 seconds) to complete. This is consistent with past experience with similar radar-sensing systems in the 5 GHz unlicensed national information infrastructure (U-NII) bands which has shown that even non-dedicated listening devices can routinely respond with positive detections within a half a second or so when exposed to radar bursts.

In practical terms, $P_{declare}$ cannot be measured directly in an efficient and effective manner to the number of significant figures (one part in one hundred, 0.01) that may be demanded; it can be more efficiently and effectively computed for each radar waveform for each ESC using collected P_{detect} data taken from those receivers.

Some supplemental direct observations of $P_{declare}$ can be taken to provide confidence that detections are indeed resulting in declarations of radar presence by an ESC. But if those observations require something like a minute per trial, there will not be sufficient testing time to obtain $P_{declare}$ with a statistical confidence of one part in one hundred (0.01) based on such observations. Such testing of $P_{declare}$ will only confirm that the declaration process normally works properly when the detection regime conditions have been satisfied.

7. REFERENCES

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APPENDIX A: NUMBER OF BINOMIAL TRIALS REQUIRED TO OBTAIN TO A GIVEN STANDARD ERROR

(The following derivation is drawn from [A-1] and the discussion has been condensed from [A-2].)

A.1 Derivation

The standard error, SE , of any set of data points, x , is a function of:

$$\sum_{i=1}^n (x_i - \bar{x})^2 \quad (\text{A-1})$$

where \bar{x} is the average of the data set.

If trial data are binomial (i.e., each trial results in either a 1 or a 0), then we count the total number of detections as q 1s and non-detections as $(n - q)$ 0s for a set of n trials.

The average of the data set will be $\bar{x} = (q/n)$ and $(x_i - \bar{x}) = (1 - q/n)$ for q trials (each counted as 1) and $(0 - q/n)$ for the remaining $(n - q)$ trials (each counted as 0). Therefore,

$$\begin{aligned} \sum_{i=1}^n (x_i - \bar{x})^2 &= q(1 - q/n)^2 + (n - q)(0 - q/n)^2 \\ &= q(1 - 2q/n + q^2/n^2) + (n - q)(q^2/n^2) \\ &= q - 2(q^2/n) + (q^3/n^2) + (q^2/n) - (q^3/n^2) \\ &= q - (q^2/n) \\ &= q(1 - q/n) \\ &= n\bar{x}(1 - \bar{x}) \end{aligned} \quad (\text{A-2})$$

The variance, V , of these trials is the square of the standard deviation, SD , of the trials. The variance is

$$V = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} = \bar{x}(1 - \bar{x}) = (SD)^2 \quad (\text{A-3})$$

SE is SD divided by the square root of the number of trials:

$$SE = \sqrt{\frac{\bar{x}(1 - \bar{x})}{n}} \quad (\text{A-4})$$

$$n = \frac{\bar{x}(1 - \bar{x})}{(SE)^2}$$

A.2 Discussion

For example, suppose that we require that SE of $P_{declare}$ must be 0.01 or less. Then the number of trials must be greater than or equal to:

$$n = \frac{\bar{x}(1 - \bar{x})}{(0.01)^2} \quad (\text{A-5})$$

Table A-1 shows how the number of trials, n , would need to vary with values of $\bar{x} = P_{declare}$ based on this analysis.

Table A-1. Example computations of n from given values of $P_{declare}$.

$P_{declare}$	0.50	0.60	0.70	0.80	0.90	0.99
n	2500	2400	2100	1600	900	99

Citizens Broadband Radio Service Devices (CBSDs); dynamic protection area (DPA); spectrum sharing; environmental sensing capability (ESC); interference monitoring; probability of detection; probability of declaration; radar; radar pulse burst; spectrum access system (SAS); spectrum sharing

If the value of $\bar{x} = P_{declare}$ were 0.90, for example, and if each trial were to take one minute, then 900 minutes would be required to complete the trials for a single data point. This is 15 hours, or two working days per data point. (A design goal might be to have $P_{declare} = 0.99$, which would only require 99 trials to complete. But test planning has to be predicated on how low the actual value $P_{declare}$ *might* be, rather than on how good it actually turns out to be.)

If on the other hand the value of $\bar{x} = P_{detect}$ (as opposed to $P_{declare}$) were 0.6 but the size of its SE only needed to be 0.1 (because in that case $P_{declare}$ can be *computed* from P_{detect} to a value that is good to 0.01), then only $((0.6) \cdot (0.4)) / (0.1)^2 = 24$ trials would be required.

A.3 References

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