

User's Guide for PRODSIR:  
A Procedure to Compute  
the Probability Distribution  
of Signal-to-Interference Ratio  
in Congested Radio Environments

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USER'S GUIDE FOR PRODSIR: A PROCEDURE TO COMPUTE THE PROBABILITY  
DISTRIBUTION OF SIGNAL-TO-INTERFERENCE RATIO IN CONGESTED RADIO ENVIRONMENTS

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PRODSIR is a procedure for computing the probability distribution of the signal-to-interference ratio in a congested radio environment. Most of the independent variables in the usual signal-to-interference equation are assumed to be random variables, and the input to the program is their probability density functions (pdfs). The pdfs can be analytical or tabular. The times and lengths of interfering emissions are assumed to be random. PRODSIR combines and transforms the input pdfs using the relationships of probability theory and produces the pdfs of the wanted signal power, the interference-plus-noise power, and the probability distribution of the signal-to-interference ratio.

PRODSIR is implemented in an interactive program. A sample session for a version of PRODSIR involving only cochannel interference and noise is included to illustrate the input and output. The structure of the program is given along with instructions for modifying it for special applications. Appendices describe some of the subroutines for handling random variables.

Key Words: interference; probabilistic EMC; signal-to-interference ratio; spectrum management

1. INTRODUCTION

In many congested spectrum-use environments, there is a significant variability in almost every factor that influences communications success. Transmitters may be mobile, or their locations may be unknown, either because data are not available or because the systems have not yet been deployed. Antenna heights may depend on the availability of favorable locations or on the desired coverage area. Transmitter powers may vary by specification or because of aging and different maintenance procedures. Transmission loss varies with details of the transmission path and, often, with time because of weather or other geophysical changes. Message traffic or channel usage may vary with the requirements of the users. Therefore, any realistic estimate of communications success must be

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probabilistic. The estimate should be based on the statistical distributions of all significant factors.

An analytical combination of all the pertinent random variables usually is impossible unless all the variables are normally distributed and simply related. The difficulty is that most combinations of random variables require integration of products of the probability density functions (pdfs). Closed-form solutions of the integrals usually not available, except for special pdfs. However, numerical integration is a simple and stable procedure, so a computer can easily combine random variables that have nearly-arbitrary pdfs. This is the motivation for PRODSIR, a procedure for computing the PRObability Distribution of Signal-to-Interference Ratio. In this report, the term PRODSIR will refer to the general procedure. Particular implementations of PRODSIR are given unique names.

Numerical integration frees the user from forcing the distributions of input random variables into some special (usually normal) form. Instead, each input pdf can be given the most realistic form. In fact, if data are available, the pdf can be a table obtained from the empirical histogram. So the input scenarios can be as realistic as available data and system descriptions allow.

Figure 1 shows a functional flow chart of PRODSIR. The analysis is given in Section 4. The situation is defined by giving the parameters of the pdfs of the input random variables, and by choosing certain control options. The wanted signal,  $S$ , is computed.

The distribution of interference power from each category of interferer is found. Interferers are in the same category if their statistical distributions are the same. Interference power is assumed to add incoherently and all interference is summed. Of course, the powers in decibels must be transformed to power in watts before adding. Similarly, the background noise power is added to the interference. When these conditional probabilities are combined with the probabilities that exactly  $n$  interferers are emitting (obtained from the traffic intensity), the result is the pdf of total interference-plus-noise.

Now the interference is subtracted from the signal (using a convolution integral) to get the pdf of the signal-to-interference ratio (SIR) (see Section 4.1). The probability that the SIR is less than a threshold  $T_n$  for a number of values of  $T_n$  is computed by integrating the pdf, and an output table is printed.

A first-generation implementation of PRODSIR was described by Berry [1977] and was applied to several Citizen's Band radio analyses [Berry, 1978a,b]. Hagn [1980] developed a program for computing the probability of communications assuming that



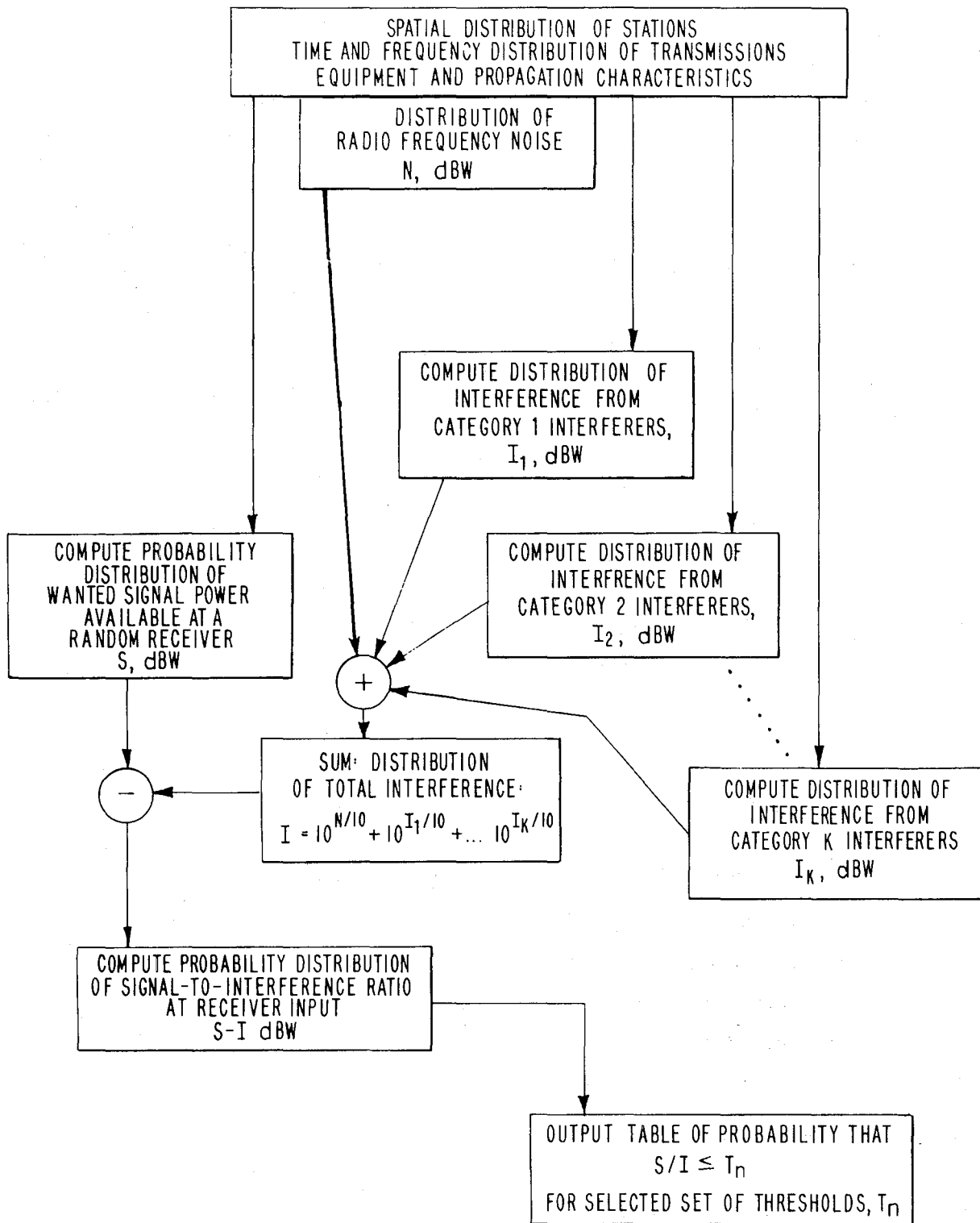


Figure 1. Functional block diagram of the PRODSIR procedure.

all random variables are normally distributed. Middleton [1979] developed an analytical solution for communications probability for a limited scenario.

PRODSIR has been implemented in an interactive FORTRAN version on a large central computer at the Department of Commerce Laboratory in Boulder, Colorado. In Section 2 of this report, an implementation called PDSIRCC for cases involving only cochannel interference and background noise is described. An adaption of PDSIRCC, called PDSIR, has been implemented on minicomputers at NTIA in Boulder, and in Annapolis, Maryland.

Section 3 shows a sample interactive session with PDSIRCC. The sample cases in the session exercise all subroutines so they can be used to verify correct installation on another computer.

The power of the numerical integration approach is its generality and flexibility. It can be applied to a wide variety of congested radio environments. Providing for all classes of interference environments within one computer program would make the program very large. A general program would also be awkward to use because of the number of inputs required and the number of options to be chosen. It is more efficient to tailor a version of PRODSIR for each class of interference environments. Section 4 contains instructions for modifying PRODSIR to fit special needs. Section 5 provides the information on the subroutines that is necessary for modifications.

The appendices A and B contain the analyses for some of the procedures. Appendix C lists run-time trouble reports and possible fixes.

## 2. THE COCHANNEL INTERFERENCE MODEL

This section describes the simplest version of PRODSIR--for a situation with only cochannel interference and background noise. The cochannel interferers are statistically homogeneous; that is, they are enough alike that their differences are covered by their probability density functions. (Methods for dealing with situations containing several categories of interferers are covered in Section 4.)

The user may want a parameter to have a fixed, deterministic value rather than a statistical distribution. If a subroutine is designed to allow this, the instructions in the following subsections tell how to do it.

Table 1 contains a brief description of the input parameters for PDSIR along with their unique indices, units and default values. The unique index makes it easy to change the value of a parameter during a session. Each parameter is an element

Table 1. Input Parameters Required by Program PDSIRCC  
 All Distances Should be in the the Same Units.

NO	DEFAULT	DESCRIPTION
1	20.00	Mean of wanted transmitter ERP, dBW. Normal in DB
2	3.00	Standard deviation of wanted transmitter ERP, dB
3	31.00	Number of points at which to define transmitter ERP
6	10.00	Ref. Dist. in wanted path length PDF
7	0.00	Offset distance in random walk PDF
8	1.00	Minimum wanted path length to be considered
9	1.00	Increment in wanted path length
10	35.00	Maximum wanted path length to be considered
11	90.00	A for loss
12	40.00	B for loss
13	5.00	Loss st. dev.
		Transmission loss is normal in dB
		Mean loss = $A + B \log_{10}(\text{path length})$
51	20.00	Mean of unwanted transmitter ERP. Normal in dB
52	3.00	Standard deviation of unwanted ERP, dB
53	31.00	No. of points at which to define unwanted ERP
61	1.00	Inner radius of annulus containing interferers
62	35.00	Outer radius. Interferers are randomly located
63	1.00	Increment in unwanted path length
Comment: Transmission loss for interferers same as for wanted signal		
101	-160.00	Mean of ambient noise, DBW. Normal in dB
102	8.00	Standard deviation of noise, dB
111	.40	Traffic intensity of interferers
112	4.00	Maximum number of simultaneous interferers
116	2.00	XINC, dB increment in PDFs. XINC > 0.5
117	3.00	PRINC. Print out PD in steps of (PRINC) X (XINC).

of a dimensioned variable called PAR in PDSIR. The following subsections describe these input parameters in detail.

The large-computer program for this model is called PDSIRCC; the minicomputer program is called PDSIR. The programs are written in FORTRAN.

## 2.1 The Wanted Signal Descriptors

The transmitter power and transmitter antenna gain have been combined into the effective radiated power (ERP) of the wanted transmitter. The ERP is assumed to be normally distributed in decibels, with a user-supplied mean and standard deviation. The number of points at which the pdf of ERP is to be defined may be specified. The default value of 31 should be used except when a single deterministic value for ERP is desired. In this case, set the mean equal to the desired ERP, set the standard deviation to zero, and set the number of points to one.

The wanted transmitter is assumed to perform a random walk about a center of operations [Middleton, 1979]. Figure 2 shows the geometry, where C is the center of operations for the mobile transmitter, and d is the random distance from the transmitter to the receiver.

Figure 3 shows three examples of the pdf of path length derived from this assumption. Four values are needed to define this pdf. The first is the "reference distance." Set this equal to 0.9 of the mean path length for the desired coverage area. The offset distance,  $d_a$ , is the distance between the desired receiver and the center of the transmitter's random walk. Often, this offset distance is zero (the transmitter operates around its base station), so the default value is zero. The other required parameters are the minimum path length to be considered, the increment in path length to be used to define the pdf, and the maximum path length to be considered. In some applications, the distance to the wanted transmitter may be fixed and known. To model this situation, set the minimum and maximum path lengths equal to the fixed distance. When this is done, the values of the other wanted path length parameters are immaterial.

The transmission loss for any fixed distance is assumed to be normally distributed in dB. The mean of the distribution depends on the distance:

$$\text{mean} = A + B \log d \quad (1)$$

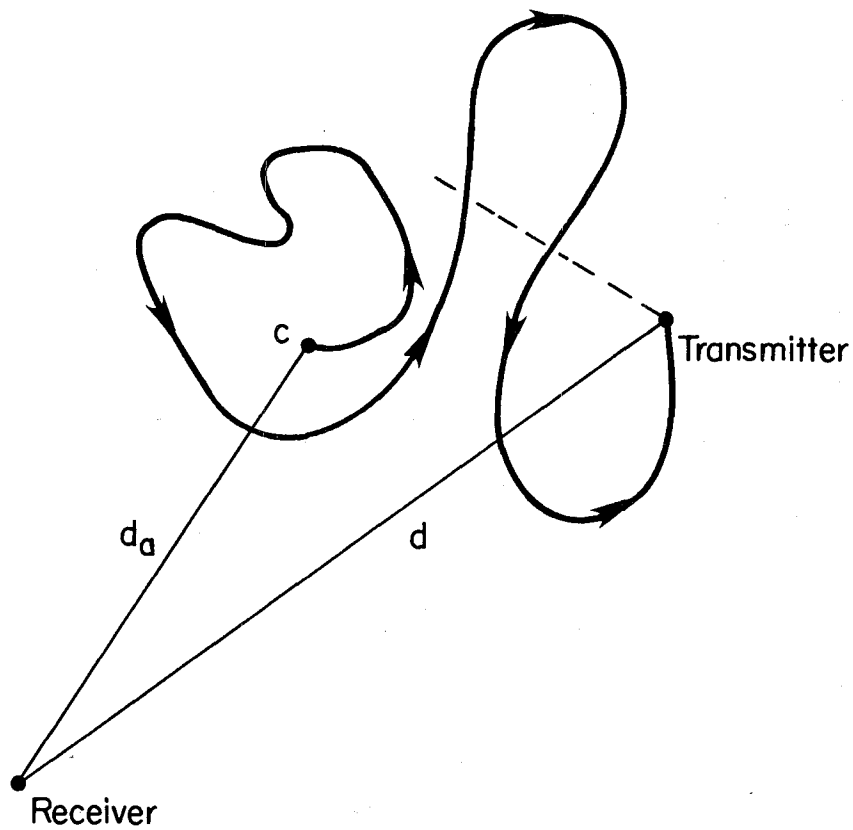


Figure 2. Geometry of wanted path for Middleton's model. The transmitter performs a random walk around its center of operations,  $C$ , which is  $d_a$  km from the receiver. The random variable  $d$  is the distance from the transmitter to the receiver.

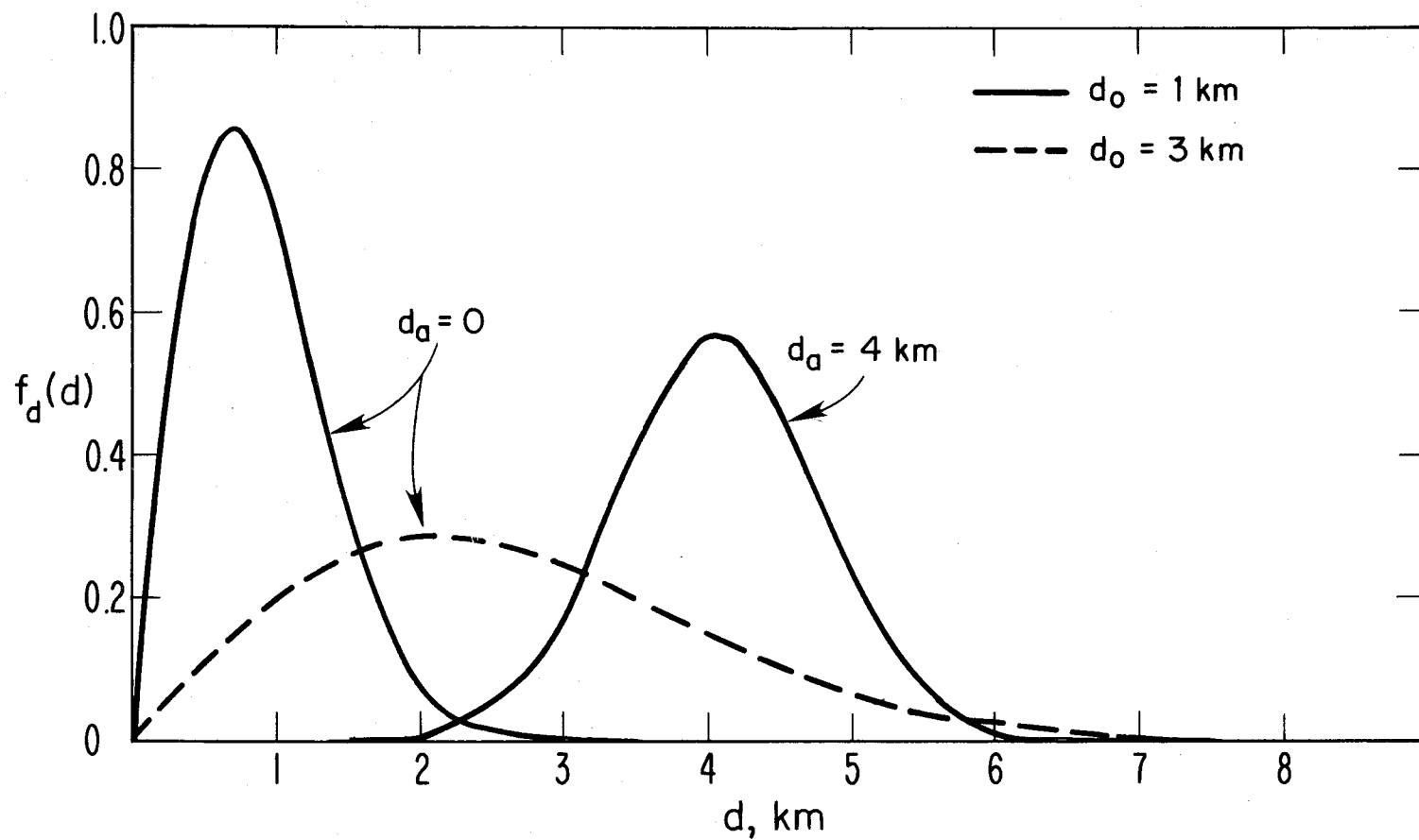


Figure 3. Examples of the wanted path probability density function. In the figure  $d_o$  is the reference distance, PAR(6), and  $d_a$  is the offset distance, PAR(7).

where  $d$  is the path length and  $A$  and  $B$  are input parameters. (In this report,  $\log$  means logarithm base 10.) The standard deviation of the transmission loss is specified in decibels. The transmitter antenna height gain, and the receiver antenna gain and height gain are included in the constants,  $A$  and  $B$ . Egli [1957] and McMahon [1974] give models that can be put in the form of (1) when the radio frequency and antenna types are known. The program can be modified to use more complicated propagation models. (See Section 4.)

## 2.2 Interference

The ERP of each interference source ("unwanted transmitter") is assumed to be normally distributed in decibels, with a user-supplied mean value and standard deviation. Transmitter power and antenna gain are included in the ERP. The sources are assumed to be randomly located in an annulus around the desired receiver. The inner and outer radii of the annulus are input parameters, as is the increment in path length used to define the pdf.

It is assumed that the interferers are similar to the wanted transmitter and are located in the same area so that the transmission loss model for interferers is the same as for the wanted signal. This may not be true in all applications. A different transmission loss model can be inserted for the interferers, as discussed in Section 4.

The interferers are assumed to emit randomly and independently, resulting in a Poisson distribution in time. The probability that there are exactly  $n$  interferers on simultaneously can then be found from the traffic intensity and the total number of potential interferers. The traffic intensity,  $U$ , is defined as the mean length of an emission times the mean number of emissions per unit time. If the number of potential emitters is small, this number is entered as the maximum number of simultaneous interferers. If the number of potential interferers is large, find a value of  $N$  such that  $U^N/N!$  is less than the error that can be tolerated, and enter this as the maximum number of simultaneous interferers.

## 2.3 Background Noise

In addition to the "interference" (coming from discrete sources), there is "background noise," which can be described by giving its probability distribution. Depending on the radio frequency and the environment, this noise could be receiver noise, galactic noise, manmade noise, or atmospheric noise. It is assumed to be normally distributed in decibels. Its mean and standard deviation are input

parameters. Empirical noise data can be found in CCIR Report 322 [CCIR, 1963], CCIR Report 258 [CCIR, 1978b], and in the book by Skomal [1978]. Section 4 tells how to substitute other noise distributions.

## 2.4 Other

The user may specify the increment in decibels to be used in defining the signal and interference distributions. It must be small enough to produce accurate numerical integration, and large enough so that the distributions fit in the storage space reserved for them. Also, calculation time increases as the step size decreases. Tests so far indicate that the increment should be between 0.5 and 5, and the default value is 2. In the early investigation of a new situation, the user should try different values for the increment to see what effect they have on the answer and on the execution time. The user can then pick a value for subsequent runs. Such experimentation is easy because the program is interactive. An example is shown in the sample session in Section 3.

The step size in the output table of the probability distribution of signal-to-interference ratio (SIR) is controlled by the user. It is an integral multiple of the increment described in the preceding paragraph. The probability that  $SIR < T_n$  will be printed at the specified intervals over the significant range of  $T_n$ .

## 3. A SESSION WITH PDSIRCC

PDSIRCC is a version of PRODSIR that includes only noise and cochannel interference. It is useful for some problems and illustrates the structure and interactive character of the method. PDSIRCC is implemented on the large computer at Boulder, and on the minicomputers at Boulder and Annapolis. The minicomputer implementation is named PDSIR, and there are some differences in its operations. These will be noted in indented paragraphs in the rest of this section.

A session at the terminal using PDSIRCC is recorded and commented on in this section. The session should be sufficient to acquaint a user with PDSIRCC. However, it is assumed that the user is not a naive terminal user, but rather has some experience at logging on, and interacting with a program. The program's instructions to the user are brief, and there are no "HELP" routines.

Because PDSIRCC requires many inputs, a default case is defined in a DATA statement in the program. A user need only define parameters that differ from the default case. If the user wants to make repeated runs, varying some parameter, only this parameter needs to be changed each time new data is requested.



In the mini-computer implementation, PDSIR, the default case is re-established at each restart. Therefore, all parameters that differ from the default case must be entered at each restart. The user can minimize data entry by redefining the default case using an editor on the source program. The default case is defined in a DATA statement in segment &SIN.

The parameters in the default case are "in the ball park" for land mobile service in the 50-70 MHz band, but have been chosen more for illustrative purposes than for realism. A user doing an extensive parametric study of a service can define a baseline case for this service and install it as the default case in the source program using an editor. For many services it will be necessary to expand the capabilities of PDSIRCC and produce a special version of PRODSIR. Section 6 contains instructions for modifying PDSIRCC for particular applications.

Figures 4 and 5 show the record of a session with PDSIRCC on the large computer in Boulder. In these figures, characters typed by the user at the terminal are underlined. Circled letters with arrows have been superimposed on the record for reference purposes.

The interaction with the program begins at circle A in Figure 4a. The input table gives the item number of each required input datum, its current value (which is the default value at this time), and a brief description of the datum. This information is needed for the next step, although the table does not need to be printed every time. Notice that this table is Table 1 of the Section 2. When the program has completed a case, it returns to circle A so that another case can be entered, if desired.

Data entry begins at circle B, Figure 4a. The default case is wanted this time, so only the "data entry complete" signal, 0,1., is entered. The user is then asked to provide case identification, and in this case, DEFAULT is entered. Program calculation begins.

The computed output begins at circle C, Figure 4b, with a heading that identifies the program and the case. A table of the means and standard deviations of the main distributions computed is printed. These can be used to check the consistency of the numerical methods. For example, the signal-to-interference ratio is computed by subtracting the interference in dB from the signal in dB. So the mean of the SIR should equal the difference of the means of the signal and the

DO YOU WANT TO SEE A LIST OF INPUT? TYPE YES OR NO.  
? YES

(A)

INPUT TABLE

NO	DEFAL	DESCRIPTION
1	20.00	MEAN OF WANTED TX ERP, DBW. NORMAL IN DB.
2	3.00	STANDARD DEVIATION OF WANTED TX ERP, DB
3	31.00	NUMBER OF POINTS AT WHICH TO DEFINE TX ERP
6	10.00	REF. DIST. IN WANTED PATH LENGTH PDF
7	0.00	OFFSET DISTANCE, ROA, IN RANDOM WALK PDF
8	1.00	MINIMUM WANTED PATH LENGTH TO BE CONSIDERED
9	1.00	INCREMENT IN WANTED PATH LENGTH
10	35.00	MAXIMUM WANTED PATH LENGTH TO BE CONSIDERED
11	90.00	A FOR TX LOSS :: TX LOSS IS NORMAL IN DB
12	40.00	B FOR TX LOSS :: MEAN LOSS =
13	5.00	LOSS ST. DEV. :: $A+B*\text{ALOG}_{10}(\text{PATH DIS})$
51	20.00	MEAN OF UNWANTED TX ERP. NORMAL IN DB.
52	3.00	ST. DEV. OF UNWANTED TX ERP, DB.
53	31.00	NO. OF PTS AT WHICH TO DEFINE UNWANTED ERP
61	1.00	INNER RADIUS OF ANNULUS CONTAINING INTERFERERS
62	35.00	OUTER RADIUS. INTERFERERS ARE RANDOMLY LOCATED.
63	1.00	INCREMENT IN UNWANTED PATH LENGTH
TX LOSS FOR INTERFERERS SAME AS FOR WANTED SIGNAL		
101-160	160.00	MEAN OF AMBIENT NOISE, DBW. NORMAL IN DB.
102	8.00	STANDARD DEVIATION OF NOISE, DB.
111	.40	TRAFFIC INTENSITY OF INTERFERERS
112	4.00	MAXIMUM NUMBER OF SIMULTANEOUS INTERFERERS.
116	2.00	XINC, DB INCREMENT IN PDFS. XINC > 0.5
117	3.00	PRINC. PRINT OUT PD IN STEPS OF PRINC*XINC

AT EACH PROMPT, TYPE PARAMETER NO., PARAMETER. (CR) EXAMPLE: 2,5. (CR)  
WHEN PARAMETERS ARE ALL DEFINED, TYPE 0,1. (CR)  
NOTE: (CR) MEANS CARRIAGE RETURN

(B)

? 0,1.  
TYPE A CASE IDENTIFIER, 10 OR FEWER CHARACTERS.  
? DEFAULT

Figure 4a. Record of session with PDSIRCC.

PDSIRCC RUN FOR CASE DEFAULT



	MEAN, DBW	STANDARD DEVIATION, DB
WANTED SIGNAL	-105.43	11.79
NOISE+INTERFERENCE	-147.44	20.05
S/(N + I)	41.98	23.26

X	PDF(X)	P(S/I<X)
-60.00	1.40E-06	.000
-54.00	5.01E-06	.000
-48.00	1.49E-05	.000
-42.00	3.83E-05	.000
-36.00	8.72E-05	.001
-30.00	1.84E-04	.001
-24.00	3.74E-04	.003
-18.00	7.45E-04	.006
-12.00	1.45E-03	.013
-6.00	2.70E-03	.025
0.00	4.58E-03	.046
6.00	6.78E-03	.080
12.00	8.52E-03	.127
18.00	9.15E-03	.180
24.00	9.05E-03	.235
30.00	9.63E-03	.290
36.00	1.21E-02	.355
42.00	1.61E-02	.439
48.00	1.93E-02	.546
54.00	1.96E-02	.664
60.00	1.68E-02	.775
66.00	1.24E-02	.862
72.00	8.03E-03	.923
78.00	4.72E-03	.961
84.00	2.52E-03	.982
90.00	1.19E-03	.993
96.00	4.86E-04	.997
102.00	1.70E-04	.999
108.00	4.91E-05	1.000
114.00	1.12E-05	1.000
120.00	1.88E-06	1.000

DO YOU WANT TO CHANGE SOME PARAMETERS? TYPE YES OR NO  
IF YOU TYPE NO, PROGRAM EXECUTION WILL TERMINATE.

? YES

Figure 4b. Record of session with PDSIRCC.

interference. The difference of the two means in Figure 1b is 42.01 dB while the mean of the difference is 41.98 dB. Thus, the numerical procedure has caused an error of 3 units in the fourth significant digit. Such an error is negligible in most applications.

The main output from the program is a table of the probability density function (pdf) and the probability distribution of  $S/(N+I)$  in Figure 4b. This ratio is denoted  $X$  in the table and is shown in column 1. Column 2 is the pdf and column 3 is the probability distribution.

Notice that the probability distribution is skewed (and, therefore, not normal). Even though the mean of  $S/(N+I) = X$  is about 42 dB, the probability that  $X < 42$  dB is only 0.439. The median of  $X$  is near 45 dB.

This direct output does not include a description of the scenario, or a listing of the input parameters. A user can keep a record of the input for each case identifier. However, the program also creates a file containing much more extensive output that can be saved or listed off-line. The file name is INFO, and its contents are illustrated later in the session.

Immediately after the output for a case is printed, the question at the top of Figure 4c is printed. The "YES" answer returns the program to data entry. No input table is necessary this time. To check whether the step size in the distributions is affecting the numerical accuracy, the increment [PAR(116)] is changed to 5 from the default value of 2. (Refer to the input table in Figure 4a and to Section 2.4.) The last line of the input table shows that if no other changes were made, the output table would be printed at increments of 15 dB. So PAR(117) is changed to 2. These entries are shown at circle D. A case identifier is entered.

Comparison of Figures 4c and 4b shows that the larger step size changed only the fourth significant digit of the answers. This is probably not important in most applications. However, an attempt was made to see the effect of using a much shorter step size. This is shown at circle E in Figure 4d, where XINC was changed to 0.5. Instead of an output table, an error message is printed out at circle F. The dimension of an array has been exceeded. The location of the problem and a trace of the calling sequence is printed out to help diagnose the cause. A listing of the source program and the information in sections 4 and 5 will help the diagnosis. It may be faster just to try different values of the input parameters.

In this case, the cause is known to be the small step size. The investigation of step size on accuracy could be continued at this time by trying a step size of, say, 1 dB. Instead, the step size is returned to the default value along with the

DO YOU WANT TO SEE A LIST OF INPUT? TYPE YES OR NO.  
? NO

AT EACH PROMPT, TYPE PARAMETER NO.,PARAMETER. (CR) EXAMPLE: 2,5. (CR)  
WHEN PARAMETERS ARE ALL DEFINED, TYPE 0,1. (CR)  
NOTE: (CR) MEANS CARRIAGE RETURN

? 116,5.  
? 117,2.  
? 0,1.

(D)

TYPE A CASE IDENTIFIER, 10 OR FEWER CHARACTERS.  
? D,XINC=5

PDSIRCC RUN FOR CASE D,XINC=5

	MEAN, DBW	STANDARD DEVIATION, DB
WANTED SIGNAL	-105.43	11.80
NOISE+INTERFERENCE	-147.39	20.08
S/(N + I)	41.96	23.30

X	PDF(X)	P(S/I<X)
-60.00	1.55E-06	.000
-50.00	1.11E-05	.000
-40.00	5.22E-05	.000
-30.00	1.87E-04	.001
-20.00	6.04E-04	.005
-10.00	1.83E-03	.016
0.00	4.60E-03	.047
10.00	8.02E-03	.111
20.00	9.15E-03	.199
30.00	9.65E-03	.292
40.00	1.46E-02	.409
50.00	1.97E-02	.583
60.00	1.68E-02	.773
70.00	9.43E-03	.906
80.00	3.87E-03	.970
90.00	1.18E-03	.993
100.00	2.39E-04	.999
110.00	2.88E-05	1.000
120.00	1.62E-06	1.000

DO YOU WANT TO CHANGE SOME PARAMETERS? TYPE YES OR NO  
IF YOU TYPE NO, PROGRAM EXECUTION WILL TERMINATE.  
? YES

Figure 4c. Record of session with PDSIRCC.

DO YOU WANT TO SEE A LIST OF INPUT? TYPE YES OR NO.

? NO

AT EACH PROMPT, TYPE PARAMETER NO.,PARAMETER. (CR) EXAMPLE: 2,5. (CR)  
WHEN PARAMETERS ARE ALL DEFINED, TYPE 0,1. (CR)

NOTE: (CR) MEANS CARRIAGE RETURN

? 116,.5

? 117,20.

? 0,1.

TYPE A CASE IDENTIFIER, 10 OR FEWER CHARACTERS.

? D,XINC=.5

DIMENSION TOO LARGE IN RVADD, NC,NCMAX = 468 400

CALLED WITH ENTRY RVSUB

CALLED FROM MAIN PROGRAM

DO YOU WANT TO CHANGE SOME PARAMETERS? TYPE YES OR NO  
IF YOU TYPE NO, PROGRAM EXECUTION WILL TERMINATE.

? YES

DO YOU WANT TO SEE A LIST OF INPUT? TYPE YES OR NO.

? NO

AT EACH PROMPT, TYPE PARAMETER NO.,PARAMETER. (CR) EXAMPLE: 2,5. (CR)  
WHEN PARAMETERS ARE ALL DEFINED, TYPE 0,1. (CR)

NOTE: (CR) MEANS CARRIAGE RETURN

? 116,2.

? 117,3.

? 111,.8

? 0,1.

TYPE A CASE IDENTIFIER, 10 OR FEWER CHARACTERS.

? D,U=.8

Figure 4d. Record of session with PDSIRCC.

default value of the print increment, PRINC, at circle G in Figure 4d. An investigation of the effect of some of the physical parameters on the results is started by doubling the value of the traffic intensity of the interferers. [Traffic intensity is PAR(111).] In an application, this increase in traffic might correspond to a doubling of the number of transmitters on a channel.

The result is shown in Figure 4e. The mean of N+I has increased about 9 dB, with a corresponding decrease in the SIR.

Reference to Section 2.1 shows that the wanted transmitter is performing a random walk around the receiver. Its location is unknown. The description of the random walk subroutine in Section 2.1, and the value of the reference distance, PAR(6), in the input table indicate that the mean path length is 9 km. What would the SIR be if the path length were known to be exactly 20 km? This can be computed by setting both the minimum wanted path length, PAR(8), and the maximum wanted path length, PAR(10), to 20, as is done at circle H in Figure 4f. The result is shown in Figure 4f. The mean wanted signal strength decreased about 17 dB, and its standard deviation is 6 dB smaller. This standard deviation provides another check on the programs accuracy, because with a fixed distance, it should equal the square root of the sum of the squares of the standard deviations of the ERP and the transmission loss, and it does.

At this point (circle I), the session was ended with a "NO" answer to the question at the bottom of Figure 4f.

In addition to the primary output shown in Figure 4, PDSIRCC stores more details and some intermediate output in a file named INFO. This file can be retrieved and examined if desired. The first 150 lines of the file for the above session are shown in Figure 5. The line identifying the program and the case is printed first, followed by the input table. This table is printed for every case in file INFO. Next the array that contains all data values is printed. The array has many more elements than required for input data for PDSIRCC. This is to allow use of PDSIRCC as a skeleton for more complicated versions of PRODSIR that may be needed for particular applications. The undefined values of the array are shown as "-I" in the parameter table. This output completely defines the input associated with the case identifier.

Figure 5b continues the listing of file INFO. The statistics of some of the intermediate pdfs computed by the program are given at the top. Then the pdf of total noise-plus-interference is listed. This is of interest because it corresponds to the pdf of Class A noise [Berry, 1981]. The pdf has two relative maxima. The

AT EACH PROMPT, TYPE PARAMETER NO.,PARAMETER. (CR) EXAMPLE: 2,5. (CR)  
 WHEN PARAMETERS ARE ALL DEFINED, TYPE 0,1. (CR)  
 NOTE: (CR) MEANS CARRIAGE RETURN

? 8,20.  
 ? 10,20.  
 ? 0,1.

(H)

TYPE A CASE IDENTIFIER, 10 OR FEWER CHARACTERS.

? A=.8,WP=20

PDSIRCC RUN FOR CASE A=.8,WP=20

	MEAN, DBW	STANDARD DEVIATION, DB
WANTED SIGNAL	-122.04	5.83
NOISE+INTERFERENCE	-138.30	21.86
S/(N + I)	16.25	22.62

X	PDF(X)	P(S/I<X)
-70.00	2.12E-06	.000
-64.00	9.95E-06	.000
-58.00	3.37E-05	.000
-52.00	9.47E-05	.001
-46.00	2.26E-04	.001
-40.00	4.74E-04	.003
-34.00	9.43E-04	.008
-28.00	1.87E-03	.016
-22.00	3.68E-03	.032
-16.00	7.01E-03	.063
-10.00	1.22E-02	.120
-4.00	1.76E-02	.210
2.00	1.95E-02	.324
8.00	1.57E-02	.432
14.00	9.53E-03	.508
20.00	6.62E-03	.553
26.00	9.45E-03	.599
32.00	1.52E-02	.673
38.00	1.81E-02	.776
44.00	1.51E-02	.878
50.00	8.68E-03	.950
56.00	3.45E-03	.985
62.00	9.42E-04	.997
68.00	1.76E-04	1.000
74.00	2.20E-05	1.000
80.00	1.66E-06	1.000

DO YOU WANT TO CHANGE SOME PARAMETERS? TYPE YES OR NO  
 IF YOU TYPE NO, PROGRAM EXECUTION WILL TERMINATE.

? NO

ADDITIONAL OUTPUT ON LOCAL FILE INFO

(I)

Figure 4e. Record of session with PDSIRCC.



PDSIRCC RUN FOR CASE D,U=.8

	MEAN, DBW	STANDARD DEVIATION, DB
WANTED SIGNAL	-105.43	11.79
NOISE+INTERFERENCE	-138.30	21.86
S/(N + I)	32.86	24.84

X	PDF(X)	P(S/I<X)
-62.00	1.74E-06	.000
-56.00	6.71E-06	.000
-50.00	2.11E-05	.000
-44.00	5.66E-05	.000
-38.00	1.34E-04	.001
-32.00	2.88E-04	.002
-26.00	5.90E-04	.005
-20.00	1.18E-03	.010
-14.00	2.29E-03	.020
-8.00	4.27E-03	.039
-2.00	7.27E-03	.073
4.00	1.09E-02	.127
10.00	1.37E-02	.202
16.00	1.47E-02	.288
22.00	1.35E-02	.374
28.00	1.17E-02	.449
34.00	1.10E-02	.517
40.00	1.20E-02	.585
46.00	1.35E-02	.662
52.00	1.39E-02	.745
58.00	1.23E-02	.824
64.00	9.40E-03	.890
70.00	6.32E-03	.937
76.00	3.83E-03	.967
82.00	2.11E-03	.984
88.00	1.04E-03	.993
94.00	4.47E-04	.997
100.00	1.65E-04	.999
106.00	5.10E-05	1.000
112.00	1.27E-05	1.000
118.00	2.37E-06	1.000

DO YOU WANT TO CHANGE SOME PARAMETERS? TYPE YES OR NO  
IF YOU TYPE NO, PROGRAM EXECUTION WILL TERMINATE.  
? YES

DO YOU WANT TO SEE A LIST OF INPUT? TYPE YES OR NO.  
? NO

Figure 4f. Record of session with PDSIRCC.

PDSIRCC RUN FOR CASE DEFAULT

INPUT TABLE

NO	DEFAL	DESCRIPTION
1	20.00	MEAN OF WANTED TX ERP, DBW. NORMAL IN DB.
2	3.00	STANDARD DEVIATION OF WANTED TX ERP, DB
3	31.00	NUMBER OF POINTS AT WHICH TO DEFINE TX ERP
6	10.00	REF. DIST. IN WANTED PATH LENGTH PDF
7	0.00	OFFSET DISTANCE, ROA, IN RANDOM WALK PDF
8	1.00	MINIMUM WANTED PATH LENGTH TO BE CONSIDERED
9	1.00	INCREMENT IN WANTED PATH LENGTH
10	35.00	MAXIMUM WANTED PATH LENGTH TO BE CONSIDERED
11	90.00	A FOR TX LOSS :: TX LOSS IS NORMAL IN DB
12	40.00	B FOR TX LOSS :: MEAN LOSS =
13	5.00	LOSS ST. DEV. :: $A+B*\text{ALOG}_{10}(\text{PATH DIS})$
51	20.00	MEAN OF UNWANTED TX ERP. NORMAL IN DB.
52	3.00	ST. DEV. OF UNWANTED TX ERP, DB.
53	31.00	NO. OF PTS AT WHICH TO DEFINE UNWANTED ERP
61	1.00	INNER RADIUS OF ANNULUS CONTAINING INTERFERERS
62	35.00	OUTER RADIUS. INTERFERERS ARE RANDOMLY LOCATED.
63	1.00	INCREMENT IN UNWANTED PATH LENGTH
TX LOSS FOR INTERFERERS SAME AS FOR WANTED SIGNAL		
101-160	8.00	MEAN OF AMBIENT NOISE, DBW. NORMAL IN DB.
102	8.00	STANDARD DEVIATION OF NOISE, DB.
111	.40	TRAFFIC INTENSITY OF INTERFERERS
112	4.00	MAXIMUM NUMBER OF SIMULTANEOUS INTERFERERS.
116	2.00	XINC, DB INCREMENT IN PDFS. XINC > 0.5
117	3.00	PRINC. PRINT OUT PD IN STEPS OF PRINC*XINC

I	PARAMETER VALUES								
1	20.00	3.00	31.00	-I	-I	10.00	0.00	1.00	
9	1.00	35.00	90.00	40.00	5.00	-I	-I	-I	
17	-I	-I	-I	-I	-I	-I	-I	-I	
25	-I	-I	-I	-I	-I	-I	-I	-I	
33	-I	-I	-I	-I	-I	-I	-I	-I	
41	-I	-I	-I	-I	-I	-I	-I	-I	
49	-I	-I	20.00	3.00	31.00	-I	-I	-I	
57	-I	-I	-I	-I	1.00	35.00	1.00	-I	
65	-I	-I	-I	-I	-I	-I	-I	-I	
73	-I	-I	-I	-I	-I	-I	-I	-I	
81	-I	-I	-I	-I	-I	-I	-I	-I	
89	-I	-I	-I	-I	-I	-I	-I	-I	
97	-I	-I	-I	-I	-I	-I	-I	-I	
105	-I	-I	-I	-I	-160.00	8.00	-I	-I	
113	-I	-I	-I	2.00	3.00	-I	.40	4.00	-I

Figure 5a. Listing of the first part of the file INFO.

AVERAGE AND S.D. OF WANTED LOSS = 125.427 11.407  
 AVERAGE AND S.D. OF WANTED SIGNAL = -105.427 11.794  
 AVERAGE AND S.D. OF UNWANTED LOSS = 143.127 9.865  
 AVERAGE AND S.D. OF 1 INT = -123.127 10.310

PDF OF TOTAL NOISE

X	PDF(X)
-192.00	1.122E-05
-190.00	2.929E-05
-188.00	7.214E-05
-186.00	1.674E-04
-184.00	3.660E-04
-182.00	7.530E-04
-180.00	1.456E-03
-178.00	2.647E-03
-176.00	4.517E-03
-174.00	7.236E-03
-172.00	1.088E-02
-170.00	1.535E-02
-168.00	2.032E-02
-166.00	2.527E-02
-164.00	2.952E-02
-162.00	3.241E-02
-160.00	3.343E-02
-158.00	3.240E-02
-156.00	2.949E-02
-154.00	2.521E-02
-152.00	2.025E-02
-150.00	1.530E-02
-148.00	1.090E-02
-146.00	7.394E-03
-144.00	4.945E-03
-142.00	3.589E-03
-140.00	3.299E-03
-138.00	3.996E-03
-136.00	5.528E-03
-134.00	7.630E-03
-132.00	9.966E-03
-130.00	1.211E-02
-128.00	1.373E-02
-126.00	1.454E-02
-124.00	1.449E-02
-122.00	1.364E-02
-120.00	1.223E-02
-118.00	1.053E-02
-116.00	8.799E-03
-114.00	7.203E-03
-112.00	5.817E-03
-110.00	4.664E-03
-108.00	3.723E-03
-106.00	2.966E-03

Figure 5b. Listing of the first part of the file INFO.

-104.00	2.360E-03		
-102.00	1.877E-03		
-100.00	1.493E-03		
-98.00	1.188E-03		
-96.00	9.432E-04		
-94.00	7.479E-04		
-92.00	5.914E-04		
-90.00	4.677E-04		
-88.00	3.718E-04		
-86.00	2.990E-04		
-84.00	2.430E-04		
-82.00	1.974E-04		
-80.00	1.572E-04		
-78.00	1.205E-04		
-76.00	8.821E-05		
-74.00	6.205E-05		
-72.00	4.281E-05		
-70.00	2.967E-05		
-68.00	2.083E-05		
-66.00	1.457E-05		
-64.00	9.821E-06		
-62.00	6.184E-06		
-60.00	3.555E-06		
-58.00	1.839E-06		
-56.00	8.493E-07		
-54.00	3.215E-07		
AVERAGE AND S.D. OF S/(N+I) =		41.983	23.260
PAR(116) =	5.000		
PAR(117) =	2.000		

PDSIRCC RUN FOR CASE D, XINC=5

#### INPUT TABLE

NO	DEFAL	DESCRIPTION
1	20.00	MEAN OF WANTED TX ERP, DBW. NORMAL IN DB.
2	3.00	STANDARD DEVIATION OF WANTED TX ERP, DB
3	31.00	NUMBER OF POINTS AT WHICH TO DEFINE TX ERP
6	10.00	REF. DIST. IN WANTED PATH LENGTH PDF
7	0.00	OFFSET DISTANCE, ROA, IN RANDOM WALK PDF
8	1.00	MINIMUM WANTED PATH LENGTH TO BE CONSIDERED
9	1.00	INCREMENT IN WANTED PATH LENGTH
10	35.00	MAXIMUM WANTED PATH LENGTH TO BE CONSIDERED
11	90.00	A FOR TX LOSS :: TX LOSS IS NORMAL IN DB
12	40.00	B FOR TX LOSS :: MEAN LOSS =
13	5.00	LOSS ST. DEV. :: A+B*ALOG10(PATH DIS)

Figure 5c. Listing of the first part of the file INFO.

first is at the mean of the background noise,  $X = -160$  dB (see input table, Figure 5a). The other is near  $X = -126$  dB. It represents the interference. The interference pdf is skewed, so the peak is not at the mean of the interference from one interferer.

Figure 5c continues the listing of INFO. After the pdf of total interference is completed, the statistics of the SIR are printed. (These were also printed in the program output.) This is the last information from the first case.

The next two lines show the data values entered at data entry for the second case. Then the contents of file INFO for the second case are listed. No more of file INFO will be shown in this report, because it is very long. In any particular application a user can refer to file INFO or dispose of it.

This session of PDSIRCC has illustrated how easily the input parameters can be varied to study their effects on the results. It has illustrated how a random variable can be set to a deterministic constant to see the effects of variability. It has showed how the program detects a problem and traces the calling sequence back to the main program to help diagnose the problem. The next two sections of the report show how to modify the program to cover situations that can not be modeled just by changing parameters in PDSIRCC.

#### 4. TAILORING PRODSIR TO A PARTICULAR APPLICATION

Many radio interference environments have characteristics not included in the program PDSIRCC, but which can be handled by the PRODSIR procedure. For example, adjacent channel interference may also be important, or the background noise may be Class A [Middleton, 1977] rather than log normally distributed. These situations can be handled by developing a version of PRODSIR using the general structure of PDSIRCC and its subroutines. That structure and the functions of the subroutines are described in detail in this section to guide the development of special versions of PRODSIR.

More detailed discussions of the analysis leading to PRODSIR are given by Berry [1977; 1978a, Appendix A].

##### 4.1 Analytical Structure for PRODSIR

The output of PRODSIR is the probability distribution of the ratio of signal to noise-plus-interference (SIR). For the purposes of this analysis, "interference" means unwanted power from discrete sources that are described by their locations,

powers, frequencies, etc. The descriptions may be probabilistic or deterministic. "Noise" means unwanted power of unspecified origin that is described by a single probability density function (pdf).

Signal and noise power can be expressed either in watts or in decibels above a watt (dBW). In PRODSIR they are expressed in decibels because some of the important input variables (e.g., transmission loss) are reported to have distributions that are nearly normal when given in decibels. Also, the calculation of the pdfs of available signal and of signal-to-interference ratio in decibels requires the subtraction of random variables, which is conveniently done with a convolution integral. On the other hand, random variables in units of decibels must be transformed to power in watts whenever power must be added.

Because the powers are expressed in decibels the signal-to-interference ratio (SIR) is found by subtraction:

$$SIR = S - UP, \text{ dB}, \tag{2}$$

where S, the wanted signal power, and UP, the unwanted power, are independent random variables. (In this report, random variables are capitalized, and  $f_X(x)$  denotes the probability density function of the random variable X.) As shown by Table 2, the pdf of SIR is found from

$$f_{SIR}(x) = \int f_S(s) f_I(s-x) ds, \tag{3}$$

where  $f_S(\bullet)$  is the pdf of the signal and  $f_I(\bullet)$  is the pdf of the noise-plus-interference.

Then the probability distribution is found by integration:

$$P[ SIR \leq R ] = \int_0^R f_{SIR}(x) dx. \tag{4}$$

#### 4.1.1 The wanted signal power

The wanted signal power is

$$S = P_w - L_w, \text{ dBW}, \tag{5}$$

Table 2. Elementary Compositions of Random Variables

(Details can be found in Zehna (1970) or other probability theory textbooks.)

Notation: If  $X$  is a random variable,  $f_X(x)$  is its probability density function (pdf).

$$f_X(x) \geq 0 \text{ if } x_1 \leq x \leq x_m; \text{ otherwise } f_X(x) = 0.$$

$$f_{XY}(x,y) \text{ is the joint pdf of } X \text{ and } Y. \quad P(X \leq t) = \int_{x_1}^t f_X(x) dx.$$

$$f_{X|Y}(x|y) \text{ is the conditional pdf of } X, \text{ given } Y.$$

Transformation: Let  $g(x)$  be a monotonic function, and let  $h(y)$  be its inverse.

$$\text{If } Y = g(X), \quad f_Y(y) = f_X(h(y)) |h'(y)|, \text{ where } h'(y) \text{ is the derivative of } h(y).$$

$$\text{In particular, if } Y = ax + b, \quad f_Y(y) = \left| \frac{1}{a} \right| f_X\left(\frac{y-b}{a}\right).$$

Arithmetic:

$$\text{If } Z = X + Y, \quad A = \max(x_1, z - y_m), \text{ and } B = \min(x_m, z - y_1), \text{ then } f_Z(z) = \int_A^B f_{XY}(x, z-x) dx. \quad \text{If } X \text{ and } Y \text{ are}$$

$$\text{independent, } f_Z(z) = \int_A^B f_X(x) f_Y(z-x) dx.$$

$$\text{If } Z = XY, \text{ then } f_Z(z) = \int_A^B \frac{1}{|x|} f_{XY}\left(x, \frac{z}{x}\right) dx. \quad \text{If } X \text{ and } Y \text{ are independent,}$$

$$f_Z(z) = \int_A^B \frac{1}{|x|} f_X(x) f_Y\left(\frac{z}{x}\right) dx. \quad \text{If } x_1 > 0 \text{ and } y_1 > 0, \text{ then } A = \max(x_1, z/y_m), \text{ and } B = \min(x_m, z/y_1).$$

$$\text{If } Z = X/Y, \text{ then } f_Z(z) = \int_A^B |y| f_{XY}(zy, y) dy. \quad \text{If } X \text{ and } Y \text{ are independent,}$$

$$f_Z(z) = \int_A^B |y| f_X(zy) f_Y(y) dy. \quad \text{If } x_1 > 0 \text{ and } y_1 > 0, \text{ then } A = \max(y_1, x_1/z), \text{ and } B = \min(y_m, x_m/z).$$

$$\text{Conditionals: If } X \text{ is conditional on } Y, \quad A = \max(x_1, y_1), \text{ and } B = \min(x_m, y_m), \text{ then } f_X(x) = \int_A^B f_{X|Y}(x|y) f_Y(y) dy.$$

where  $P_w$  is the wanted transmitter power and  $L_w$  is the transmission loss between the wanted transmitter and the receiver. In general, transmission loss,  $L$ , is found from

$$L = L_b - G_t - G_r, \quad \text{dB}, \quad (6)$$

where  $L_b$  is the basic transmission loss and  $G_t$  and  $G_r$  are the gains of the transmitting and receiving antennas, respectively [CCIR, 1978c]. If the pdfs for the random variables on the right in (6) are known, the pdf of  $L$  can be found by two successive convolutions such as (3). The pdfs of  $G_t$  and  $G_r$  are input, or they may be constants that are combined with  $P_w$  or  $L$ .

Propagation models usually give  $L_b$  as a conditional distribution,  $f(L_b | R, H_t, H_r)$ . (The notation  $f(X|A, B)$  means the pdf of  $X$ , given  $A$  and  $B$ .) Here,  $R$  is the path length or the distance between the transmitter and the receiver, and  $H_t$  and  $H_r$  are the heights of the transmitting antenna and the receiving antenna respectively. For example, data may show that the transmission loss is normally distributed in decibels with a mean and standard deviation that depend on distance and the antenna heights. The phrase, "that depend on," translates to the conditional pdf. Table 2 shows how to find the pdf for the unconditional loss:

$$f_{L_b}(x) = \iiint f_{L_b}(x|r, h_t, h_r) f_R(r) f_{H_t}(s) f_{H_r}(t) dt ds dr. \quad (7)$$

The pdfs of the path length,  $R$ , and the antenna heights,  $H_t$  and  $H_r$ , are input determined by the situation, but any or all may be constant. It is also necessary to define the conditional pdf of transmission loss. Several propagation models are available for this [Egli, 1957; McMahan, 1974; CCIR, 1978a, Rec. 37 and Rep. 567; Hufford, Longley, and Kissick, 1982].

In PDSIRCC, the antenna heights and antenna gains are taken to be constant. Transmitter antenna gain is combined with the transmitter power into the effective radiated power. Receiver antenna gain and the height gains are included in transmission loss. Similar combinations may be desirable in many situations; the user makes such decisions using engineering judgement.

The pdf of path length  $R$  depends on the geographic location of the wanted receivers and transmitters. In a fixed service, this path length may be known, so that  $R$  is a constant. In a mobile service, or in a system analysis of a service yet to be deployed, the path length may be probabilistic, so its pdf must be entered.



There are two path length pdf subroutines provided with PDSIRCC. One is the pdf for a random walk [Middleton, 1979], and the other is for sources distributed randomly in a circle (see 5.2.2 and 5.2.5). A user can substitute subroutines that fit a given situation better.

#### 4.1.2 The unwanted interference power

The unwanted signal power is the total of the background noise and the power from all categories of interferers. It is necessary to have more than one category of interferer whenever differences in some characteristic of the interferers are so great that the characteristic cannot be modeled with a single probability density function. For example, local and skywave interferers have to be treated as two categories because the transmission loss for ground wave propagation is so different from the transmission loss for sky wave propagation [Berry, 1978a]. Different categories also are necessary if the functional relationships are different. For example, cochannel and adjacent channel interferers must be in different categories because the adjacent channel rejection by the receiver must be subtracted from adjacent channel interference, but not from cochannel interference.

The total unwanted signal power,  $UP'$ , in watts, is

$$UP' = N' + I_1' + I_2' + \dots \quad (8)$$

where the sum on the right includes all categories of interferers. Before the noise and interference can be added, they must be converted from decibels above a watt to watts. The required transformation can be derived from Table 2. Let  $Y = 10^{(X/10)}$ , so  $Y$  is in watts if  $X$  is in decibels above a watt. Then the pdf of  $y$  is

$$f_Y(y) = (10 \log e) f_X(10 \log y)/y \quad (9)$$

When the powers have been summed, the sum must be transformed back to decibels for subsequent use in (2). So let  $W = 10 \log Z$ . Then

$$f_W(w) = ((\ln 10)/10) 10^{(w/10)} f_Z(10^{(w/10)}), \quad (10)$$

where  $\ln x$  is the natural logarithm of  $x$ . This transformation gets the pdf of  $UP$  from the pdf of  $UP'$ . Appendix B describes a subroutine for transformations (9) and (10), and for doing the addition in (8).

"Noise" includes any unwanted power than can be described by a single pdf. In PDSIRCC, the noise is assumed to be normally distributed in decibels, with the mean and standard deviation entered by the user. Input values for atmospheric, manmade, and galactic noise can be found in CCIR reports, or may have been measured for a particular locality. If the noise is known to have a non-normal distribution, such as exponential, Class A, or Class B [Middleton, 1977], the user can write a subroutine for this pdf, and use it. Sections 5.3.1 and 5.3.2 are examples of substitute subroutines for noise.

The pdf of interference from each category of interferer is computed with the same general procedure, although the input pdfs will be different in at least some respects, and the number of factors considered may vary. Let  $I_j$  be the interference power from the  $n$ th category of interferer. Then the pdf of  $I_j$  is [Berry, 1980]

$$f_{I_j}(x) = \sum_{k=0}^N f_{I_j|k}(x | k \text{ interferers on}) P[k \text{ interferers are on}], \quad (11)$$

where  $N$  is the number of potential interferers in the category.

The probability that  $k$  interferers are on depends on the circuit discipline for cochannel interferers, and on the number of potential interferers and duty cycle (traffic intensity) in other cases. In PDSIRCC, it is assumed that transmitter emissions are independent, and that the length of emissions is independent. This leads to a poisson distribution of emissions, so that the probability that there are no interferers emitting is [Kleinrock, 1975]

$$P[0 \text{ on}] = \left( \sum_{n=0}^N U^n/n! \right)^{-1}, \quad (12)$$

where  $U$  is the traffic intensity, the product of the average length of an emission and the average number of emissions per unit time. If  $N$  is infinite, then

$$P[0 \text{ on}] = \exp(-U), \quad (13)$$

and (13) is a good approximation to (12) if  $U^N/N! \ll 1$ . Then,

$$P[k \text{ on}] = (U^k/k!)P[0 \text{ on}], \quad k=1, N. \quad (14)$$

In a disciplined mobile service, cochannel stations will not transmit independently. If both discipline and information are perfect, the probability that two or more cochannel transmitters are on simultaneously is zero. However, a user's information may not be perfect -- he may not hear another station even if it is on-- so  $P[k \text{ on}]$  may not be zero, even if it is very small. Different models for  $P[k \text{ on}]$  can be substituted by the user. The discussion of Subroutine TOTPD in Section 5.1.10 tells how to do this.

To compute the pdf of interference, given that there are exactly  $k$  interferers on, the pdf of interference for exactly one random interferer is computed first. The formula is:

$$I_{j|1} = P_j - L_j - J_j - A_j, \quad (15)$$

where  $I_{j|1}$  is the interference power from the exactly one interferer  
 $P_j$  is the transmitter power,  
 $L_j$  is the transmission loss,  
 $J_j$  is the frequency-dependent reflection of the receive to the emission, and  
 $A_j$  is the processing gain of the receiver for the emission, all for interferers in the  $j$ th category.

In many cases, especially for cochannel interferers of the same kind as the wanted transmitter,  $J_j$  and  $A_j$  will be zero. Or they may be assumed to be deterministic constants and combined into a single constant. The transmission loss,  $L_j$ , is computed in the same way as it is for the wanted signal, although at least some of the inputs that determine it are presumably different.

Equation (15) is for the interference power from a random interferer, given that it is emitting. The interfering power from  $k$  simultaneous interferers from category  $j$  is also needed. It is here that the statistical homogeneity of the interferers in a category is needed. If all interferers have the same probability density function (this is what defines a category), the interference from  $k$  simultaneous interferers is the sum of  $k$  identically distributed random variables. (Before summing, the power in decibels must be transformed to power in watts.) Subroutine POWSUM (described in Appendix B) performs the transformation and takes the sum.

When (11) has been evaluated for each category of interferer, the total noise and interference is found by summing all unwanted power, using the subroutine POWSUM, and (2) gives the signal-to-interference ratio.

#### 4.2 Changing the Specification of a Controllable Random Variable

In this section, it is assumed that the user has a listing of the source program, PDSIRCC. A listing can be obtained from the author.

One way to implement the PRODSIR procedure for a particular application is to modify the program PDSIRCC. For example, the user may want to change the form of the pdf of an input random variable. This has been mentioned several times in Section 4.1. To do this, the user should:

- (i) Write a subroutine for the desired pdf,
- (ii) Change the call in the appropriate place in the main program,
- (iii) Change the "input table" description in FORMAT statements 3 and 4, and
- (iv) Change the name of the program to avoid later confusion.

Section 5 describes the form and functions of the PDSIRCC subroutines, and of some available substitute routines. The user can use an appropriate subroutine as a guide for producing a replacement. In general, a random variable in the program involves three program variables: the random variable array, the probability density array, and the number of elements defined in these two arrays. The three variables are always listed in this order in calls to the subroutines.

The substitute subroutine may require more or different input parameters. All input parameters are stored in the dimensioned variable named PAR, which is much larger than the number of input parameters required by PDSIRCC, so there is plenty of space for additional parameters. Choose unused elements of PAR for the new input values required, and add descriptions of these values to the "Input Table" in FORMAT statements 3 and 4. Add the subscripts of the chosen PAR values to array IP3 or IP4 as appropriate, and change the values of IP3END and IP4END. It is desirable to define default values for the new input parameters and to add these default values to the DATA statement.

One way to define a pdf is to enter in a table of values  $(x, \text{pdf}(x))$  derived from a histogram of empirical data.

The call to the replacement subroutine should be inserted in the program at the appropriate place. To avoid confusion, a new program name should be put in the PROGRAM statement, and in FORMAT statement 22, which is the heading of the output.

Special care must be taken when changing the transmission loss model. The probability density of transmission loss at a given power and a given distance is computed in a FUNCTION subroutine. The FUNCTION name must be declared EXTERNAL in the main program, and must be an input parameter to Subroutine TLOS. Any parameters it requires (other than power and distance) must be in COMMON /TLOSS/. In addition, it must have a provision to compute the smallest and largest values of transmission loss that are significant. See the description of function TLNOR in Section 5.2.3 for an example.

#### 4.3 Adding a New Category of Interferers

A more extensive modification is necessary to add a new category of interferers. In PDSIRCC, the noise is added to interference from a single interferer after that pdf had been computed. Then the interference from exactly  $k$  interferers is computed in the same loop as the sum in (11) is accumulated. When there are two or more categories of interferers, the noise must be added last.

For each category of interferer, compute the pdf of interference for exactly one interferer, (15), as it is done in PDSIRCC, except that steps to subtract the frequency-dependent rejection and processing gain may need to be added. (Use Subroutine RVSUB.) Then, before adding the noise, compute the pdf of interference from exactly  $k$  interferers (using Subroutine POWSUM) and accumulate the sum in (11) (using Subroutine TOTPD, section 5.1.10) in a loop.

Repeat this procedure for each category of interference. Add the different interference powers together using Subroutine POWSUM. Then call Subroutine TOTPD with the noise pdf and the total interference pdf with  $F_1$  equal to the probability that no interferer of any category is on, and  $F_2 = 1 - F_1$ . These probabilities will be determined by the probabilities that different categories of interferers are on.

#### 4.4 Changing the Service Protocol

The probability that there are  $k$  interferers on simultaneously depends on the operating rules and courtesy in the service of interest. For example, in the Fixed and Broadcasting services, transmitters are on virtually all the time, so  $P[k \text{ on}] = 1$ , if there are  $k$  transmitters in the channel, and  $P[k \text{ on}] = 0$  for any other  $k$ . In a disciplined land mobile service, there is very low probability that

there are any cochannel interferers, and very low probability that there is more than one interferer per channel in the non-cochannel channels. In an undisciplined service (Citizens Band?), it may be assumed that transmitters emit independently. This is the assumption made in PDSIRCC. It may also be assumed that transmitters beyond a certain range emit independently. If the sources of interference are not communications equipment (for example, automobile ignitions), then the emissions are probably independent and poisson. The user must define the best way to model the probability of  $k$  simultaneous interferers in each category.

These probabilities are used in the call to Subroutine TOTPD, which accumulates the sum in (11) (see Section 5.1.10). In the first call to Subroutine TOTPD,  $F1$  is the probability that there are no interferers on and  $F2$  is the probability that there is exactly one interferer on. In the  $k$ th call to Subroutine TOTPD for a given category of interferer, set  $F1=1$  and  $F2 = P[ k \text{ interferers are on } ]$ .

## 5. PRODSIR SUBROUTINES

The subroutines used in PDSIRCC are divided into two classes in this section. The first set of subroutines are general subroutines for handling probability density functions (pdfs) numerically. These subroutines are described in section 5.1. Subroutines in the second set generate the pdfs of certain random variables given the necessary parameters. Section 5.2 contains descriptions of the second set of subroutines. Section 5.3 contains descriptions of available subroutines that can be used for developing a program to fit a particular application.

There are three variables associated with each pdf: the random variable array, the pdf array, and the number of elements defined in the first two arrays. They are always in that order in the subroutine calls.

Most of the subroutines check the number of elements being defined against the maximum dimension of the array. If the dimension is exceeded, a warning is printed, and an alternate return is made that traces the calling sequence back to the main program. The user can then change the input and restart the program.

If the subroutine requires temporary array storage space, the space used is in COMMON / PAD/ ( "a scratch pad"). In addition, most subroutines reference COMMON /ZE/, which contains a single variable, ZERO. ZERO is the error tolerance used for consistency throughout the program. It is set in the main program. It should not be smaller than the least significant digit for a single-precision, real variable for the computer used. That is, if a computer provides six significant decimal

digits for a single-precision real variable, the ZERO should be at least  $10^{-6}$ . It may be even larger, if desired.

If the name or parameter list of a Subroutine is different in the minicomputer version (PDSIR), it is shown as the last line of the subsection title. For the large computer, TRAC is the statement label of the alternate return when trouble is encountered. For the minicomputer, the statement label of the alternate return is put in the position of the asterisk.

## 5.1 Subroutines for Handling Probability Density Functions

### 5.1.1 SUBROUTINE AVSIG(X,PDFX,NX,AVX,SIGX), RETURNS (TRAC)

SUBROUTINE AVSIG(X,PDFX,NX,AVX,SIGX,\*)

AVSIG computes the mean and standard deviation of an arbitrary pdf.

Input: X is an evenly spaced random variable, PDFX is its pdf, and NX is the number of values defined for each.

Output: AVX is the mean and SIGX is the standard deviation of X. If  $NX=1$ ,  $AVX=X(1)$  and  $SIGX=0$ . Otherwise, AVX and SIGX are computed using subroutine QUAD1 for numerical integration.

### 5.1.2 SUBROUTINE CHINC(X,FX,NX,DEL,XI,FXI,NXI,IMAX), RETURNS (TRAC)

SUBROUTINE CHINCI(X,FX,NX,DEL,XI,FXI,NXI,IMAX), RETURNS (TRAC)

SUBROUTINE CHINC(X,FX,NX,DEL,XI,FXI,NXI,IMAX,IOPT,\*)

Subroutine CHINCI is an alternate entry to Subroutine CHINC. The subroutine changes the increment (the spacing) in a random variable and its pdf.

Input: X is random variable, FX is its pdf, and NX is the number of points. DEL is the increment desired in the output random variable, XI, and IMAX is the dimension of the arrays in the call. TRAC is the alternate return if the dimension is exceeded.

For the minicomputer, there is only one entry. IOPT=1 is entry CHINC, and IOPT=2 is entry CHINCI.

Output: XI is the output random variable, spaced at increment, DEL. FXI is the pdf of XI, found by interpolation in FX. NXI is the number of points defined in XI and FXI. For entry CHINC, the range of XI is centered on the range

of X. For entry CHINCI, the XI are integer multiples of DEL, with XI(1) being the integer multiple of DEL nearest X(1).

**Note:** The input and output arrays can have the same name in the calling program, so CHINC can be used to change the spacing of an array and its pdf without using any extra storage space.

5.1.3 SUBROUTINE INTERP(IWARN,X,FX,NX,Y,FY,NY)  
SUBROUTINE INTEP (IWARN,X,FX,NX,Y,FY,NY)

**Input:** IWARN is a warning printout option. If IWARN is non-zero, a warning is printed out if a value of Y is outside the range of X. If IWARN=0, no warning is printed. In either case, the interpolation is carried out. X is an array, and  $X(i) > X(j)$  if  $i > j$ . FX is a non-negative function of X, and NX is the number of points.

**Output:** If  $NX=1$ ,  $FY(i)=FX(1)$ ,  $i=1,NY$ . If  $NX=2$ ,  $FY(i)$  is the value of FX at  $Y(i)$ , found using linear interpolation, for  $i=1,NY$ . If  $NX>2$ ,  $FY(i)$  is found using quadratic interpolation, provided the result is positive. If quadratic interpolation results in a negative value, and if there are values of X on both sides of  $Y(i)$ , then linear interpolation is used; otherwise,  $FY(i) = 0$ . This is because a pdf must be non-negative.

**Note:** The  $X(i)$  need not be evenly spaced, but if  $X(2)-X(1)=X(NX)-X(NX-1)$ , the formula for evenly-spaced values will be used.

5.1.4 FUNCTION QUAD1 (X,F,N)

QUAD1 is the integral of the function F from X(1) to X(N).

**Input:** X is an evenly-spaced, monotonic array, and F is a function of X. N is the number of points defined in X and FX.

**Output:** If  $N=1$ ,  $QUAD1=0$ . Otherwise, QUAD1 is the integral of F from X(1) to X(N). Quadratic integration is used if N is odd. Otherwise, quadratic integration is used for all but the last interval, where linear integration is used.



5.1.5 SUBROUTINE POWSUM(N1,N,TU,FTU,NU,TV,FTV,NV,DW,W,FW,NW,NWTOP), RETURNS (TRAC)  
SUBROUTINE POWSM (N1,N,TU,FTU,NU,TV,FTV,NV,DW,W,FW,NW,NWTOP,\*)

POWSUM transforms random variables and their pdfs from dB to power, adds them, and converts the sum back to dB. Let  $X = 10^{(TU/10)}$ , and  $Y = 10^{(TV/10)}$ . A random variable Z is found as described below, and then  $W = 10 \log Z$ . A more complete description of the problem and the analysis on which POWSUM is based are given in Appendix B.

Input: N1 is a control variable. If N1 is not positive, then Z is the sum of X and N random variables with the same distribution as Y. If N1=1, Z=X+Y. If N1>1, one more random variable with the pdf of Y is added to Z. TU is an evenly spaced array, FTU is its pdf, and NU is the number of points in TU and FTU. TV is an evenly-spaced array, FTV is its pdf, and NV is the number of points in TV and FTV. DW is the dB spacing to be used in the output array, W. NWTOP is the dimension of the input arrays.

Output: W is an array with spacing DW, FW is its pdf, and NW is the number of points defined in W and FW. W has been calculated as described above.

There are four subroutines called by POWSUM that are not general purpose. They exist only for POWSUM, and their functions can be understood only in conjunction with Appendix B. Subroutine GO computes the upper and lower limits, and associated indices for a special integral in POWSUM. Function SPINT computes that integral. Subroutine RENORMR (called RENMR in PDSIR) renormalizes a pdf in the special form required by SPINT so that its integral over its range is exactly one. See explanation in 5.1.6 below. Function SLOG computes the natural logarithm of  $(Z-X1)/(Z-X2)$  accurately even if Z is many orders of magnitude larger than X1 and X2.

5.1.6 SUBROUTINE RENORME (X,PDF,N)  
SUBROUTINE RENME (X,PDF,N)

This subroutine renormalizes a probability density function so that the integral over its range is one, as required by probability theory. It corrects for inaccuracies introduced by the approximate numerical methods used to manipulate the pdfs.

Input: X is a random variable array, evenly spaced. PDF is an array, the pdf of X. N is the number of points in X and PDF.

Output: PDF is replaced with new values so that the integral of PDF from X(1) to X(N) is one. To be accurate,  $QUAD1(X,PDF,N) = 1$ .

5.1.7 SUBROUTINE RVADD (XA,A,NA,XB,B,NB,XC,C,NC,NCMAX), RETURNS (TRAC)  
SUBROUTINE RVADD (IADD,XA,A,NA,XB,B,NB,XC,C,NC,NCMAX,\*)

This subroutine adds two random variables using numerical integration of the convolution integral of their pdfs. Either or both of the "random variables" can be constant. Appendix A contains the analysis used to develop Subroutines RVADD and RVSUB.

Input: XA is an evenly-spaced array, A is its pdf, and NA is the number of points in XA and A. XB is an evenly spaced array, B is its pdf, and NB is the number of points in XB and B. NCMAX is the dimension of the arrays in the call.

In the minicomputer version IADD=1 means add XA and XB; IADD=-1 means subtract XB from XA.

Output: XC is an evenly-spaced array. It is the random variable,  $XC = XA + XB$ , and C is its pdf. NC is the number of points.

In the minicomputer version,  $XC = XA + IADD \cdot XB$ .

Note: The increment in XC will be the smaller of the increment in XA and the increment in XB, unless this causes NC to be greater than NCMAX. In this case, the increment in XC is some power of two times the smaller input increment.

5.1.8 SUBROUTINE RVSUB (XA,A,NA,XB,B,NB,XC,C,NC,NCMAX)

Subroutine RVSUB is actually an alternate entry of Subroutine RVADD, so the input is exactly the same, and the same Note applies. However, the output is the difference of the two random variables.

Output: XC is a random variable,  $XC = XA - XB$ . C is the pdf of XC.

See Appendix A for the analysis for RVSUB.

5.1.9      SUBROUTINE SHRINK (X,ARRAY,N), RETURNS (TRAC)  
            SUBROUTINE SHRIK (X,ARRAY,N,\*)

This subroutine shrinks the range of a random variable to its significant range by deleting tails with probability that is practically zero. This is useful because a series of numerical additions or subtractions of random variables will build up formally a very large range of the resultant random variable, even though the probability density near the ends of this range is very small. If nothing is done about this, the storage and computation time required become impractical, and most of them are wasted. To prevent this, SHRINK is called after most of the manipulations. SHRINK estimates an upper bound on the integral under the tail of a pdf. If the bound is smaller than the variable ZERO (the error tolerance used throughout the program, see beginning of Section 5) the tail is eliminated. Then the values of X and ARRAY are shifted so that the significant values of the pdf begin at [X(1), ARRAY(1)], and N is decreased. Then the bounds are estimated for the right tail of the pdf, and N is decreased again.

Input:    X, a random variable array with pdf ARRAY. N is the number of points in X and ARRAY.

Output:   X, ARRAY, and N, with N possibly smaller than before. If so, X, and ARRAY have been renumbered to cover only the significant range.

5.1.10     SUBROUTINE TOTPD (X,FX,NX,Y,FY,NY,F1,F2,Z,FZ,NZ,NZMAX), RETURNS (TRAC)  
            SUBROUTINE TOTPD (X,FX,NX,Y,FY,NY,F1,F2,Z,FZ,NZMAX,\*)

This subroutine is used to accumulate the sum in (11), one term per entry. It adds  $F1 \cdot FX$  to  $F \cdot FY$ .

Input:    Evenly-spaced array X, its pdf FX, and NX, the number of points in X. Y, which has the same spacing (but not necessarily the same range) as X, its pdf, FY, and NY the number of points in Y. F1 and F2 are numbers between zero and one, and NZMAX is the dimension of the arrays in the call.

Output:   Z, an array which spans X and Y and has the same spacing.  $FZ(i) = F1 \cdot FX(Z(i)) + F2 \cdot FY(Z(i))$ , where  $FX(Z(i)) = 0$  if Z(i) is outside the range of X, and  $FY(Z(i)) = 0$  if Z(i) is outside the range of Y. NZ is the number of points defined in Z and FZ.

## 5.2 Subroutines for Producing Desired Probability Density Functions

This section describes the subroutines called by PDSIRCC to produce the pdfs of input random variables such as effective radiated power. Many of these subroutines have the option of returning a constant rather than a pdf so a program calling them can explore the effect of parameter variability on SIR.

These subroutines can be used as models for developing subroutines for other probability distributions. Often, only the lines computing the functional value will need to be changed.

### 5.2.1 SUBROUTINE NORPDF (AV,SIG,NPTS,NSIG,X,PDFX,NMAX) SUBROUTINE NORPDF (AV,SIG,NPTS,NSIG,X,PDFX,NMAX)

This subroutine computes a truncated normal distribution for given mean and standard deviation. The range of the distribution is specified in standard deviations, and the output pdf is normalized so that its integral is one.

**Input:** AV is the mean and SIG is the standard deviation of the output pdf. NPTS is the number of points at which the pdf is to be defined. The output random variable will range from NSIG standard deviations below the mean to NSIG standard deviations above the mean. NMAX is the dimension of X and PDFX in the calling program.

**Output:** If NPTS = 1, X(1) = AV, and PDFX(1) = 1. Otherwise, X is an evenly spaced array centered on AV and ranging from AV - NSIG\*SIG to AV + NSIG\*SIG. PDFX is the pdf of a truncated normal distribution. That is, PDFX has been computed first using the formula for the probability density of a normal distribution, then normalized so that its integral over the range of X is one. The difference is trivial if NSIG > 3.

### 5.2.2 SUBROUTINE PLAN (RBEG,RINC,REND,X,WPAL,N,NMAX), RETURNS (TRAC) SUBROUTINE PLAN (RBEG,RINC,REND,X,WPAL,N,NMAX,\*)

This subroutine computes the pdf of distances (path lengths) from the center of an annulus to random points in the annulus. "Random" means that the probability that an area contains a point is proportional to the size of the area.

Input: RBEG is the inner radius of the annulus, and REND is the outer radius. RBEG  $\leq$  REND. RINC is the increment in the output array, X. NMAX is the dimension of X and WPAL in the calling program.

Output: If RBEG=REND, then N=1, X(1)=RBEG, and WPAL(1) = 1. Otherwise, X is an array with spacing RINC and range [RBEG,REND]. WPAL is the pdf of X and N is the number of points defined.

### 5.2.3 FUNCTION TLNOR (X,D,IGO)

This function returns the probability density for transmission loss X at a given distance D when transmission loss is normal in dB. It is also used to compute the range of transmission loss to be considered. Some of its input is in COMMON / TLOSS /.

Input: X, transmission loss in DB, and D, a distance. IGO indicates which of three outputs is desired; IGO = -1, 0, or 1. A (in common) is a constant in the formula for the transmission loss mean. B (in common) is the transmission loss growth rate with distance. SIG (in common) is the standard deviation of transmission loss.

Note: The mean transmission loss at distance D is given by equation (1).

Output: If IGO = -1, TLNOR is the minimum transmission loss for distance D. It should be used with D equal to the minimum path length. If IGO = 1, TLNOR is the maximum transmission loss for distance D. It should be used with D equal to the maximum path length. If IGO = 0, TLNOR is the probability density at (X,D). It is a normal pdf with the mean given in the note above, and with standard deviation equal to SIG.

### 5.2.4 SUBROUTINE TLOS (D,PL,ND,XINC,TXL,T,PT,NT,NIMAX), RETURNS (TRAC) SUBROUTINE TLOS (D,PL,ND,XINC,TXL,T,PT,NT,NIMAX,\*)

This subroutine computes the pdf of transmission loss given the pdf of path lengths and the conditional transmission loss (see (7)).

Input: D is an array of evenly-spaced path lengths, PL is its pdf, and ND is the number of points defined in D and PL. XINC is the increment wanted in the output array T. TXL is the name of the function subroutine that will calculate the probability density of transmission loss at a given distance

and transmission loss. It is the conditional pdf in (7). NITMAX is the dimension of the arrays in the call.

Output: T is the array for the random variable, transmission loss, spaced at increments of XINC. PT is the pdf of T, and there are NT points defined in T and PT.

5.2.5 SUBROUTINE WPLRW (XOA, REF, RBEG, RINC, REND, X, WPAL, N, NMAX), RETURNS (TRAC)  
SUBROUTINE WPLW (XOA, REF, RBEG, RINC, REND, X, WPAL, N, NMAX, \*)

This subroutine computes the pdf of path lengths in a non-central random walk [Middleton, 1979]. It calls FUNCTION SI0(X), which is  $\exp(-X)$  times the modified Bessel function  $I_0(x)$  [Abramowitz and Stegun, 1964]. Figure 2 shows the geometry and Figure 3 is a plot of this pdf for different parameters.

Input: XOA is the non-central displacement, the distance from the origin to the center of operations of the mobile. REF is the reference distance, a scaling factor. The mean path length will be about 0.9 REF. RBEG is the shortest path length, i.e.,  $X(1)=RBEG$ . REND is the longest path length, and RINC is the increment in path length. RINC will be spacing in the output array, X. NMAX is the dimension of the X and WPAL in the calling program.

With XOA and REF determined from the physical situation, suitable choices of the other parameters are:

- a. if  $XOA < 0.5$ ,  
 $RINC = REF/10$ ,  $RBEG = .007(REF)$  and  $REND = 4(REF) + XOA$
- b. if  $XOA > 0.5$ ,  
 $RINC = REF/5$ ,  $RBEG = \text{Max}(REF/100, XOA - 4.(REF))$  and  
 $REND = XOA + 4.(REF)$ .

Output: IF  $RBEG = REND$ , then  $N = 1$ ,  $X(1) = RBEG$ , and  $WPAL(1) = 1$ . Otherwise X is the array of path lengths, and WPAL is the pdf of X. N is the number of points defined in X and WPAL.

### 5.3 Additional Subroutines

This section describes subroutines which can be used to implement other versions of PRODSIR.

#### 5.3.1 SUBROUTINE BIMOD (AMU1,SIGMA1,W1,AMU2,SIGMA2,XINC,X,POW,NX)

This subroutine produces a bimodal normal probability density function; that is, it produces a double-humped pdf such that either hump has the shape of a normal pdf. The relative sizes of the humps are determined by a weight, W1.

**Input:** AMU1 is the location of the peak of one hump, and AMU2 is the location of the peak of the other hump. SIGMA1 is a measure of the spread of the first hump. It would be the standard deviation of the normal distribution if W1=1. SIGMA2 is a measure of the spread of the second hump. It would be the standard deviation of the normal distribution if W1=0. W1 is the weight of the first hump,  $0 \leq W1 \leq 1$ . XINC is the increment in the output array, X.

**Output:** X is an array with spacing XINC, POW is its pdf, and NX is the number of points at which X and POW are defined.  $POW(X) = W1 * NOR(AMU1, SIGMA1) + (1 - W1) * NOR(AMU2, SIGMA2)$ , where NOR(A,S) is a normal pdf with mean, A, and standard deviation, S.

#### 5.3.2 SUBROUTINE CLASA (AV,AA,GR,DELTA,X,PDFX,N)

This supplementary subroutine computes the pdf of Middleton's canonical Class A noise power [Middleton, 1977], for the power in dBW.

**Input:** AV is the mean of the noise, in watts. AA is Middleton's parameter  $A_A$ . It is equivalent to the traffic intensity, U, in Section 5. GR is Middleton's parameter  $\Gamma_A$ . It is the ratio of the mean of the gaussian noise component to the mean of the non-gaussian noise component. DELTA is the spacing in the output array, X.

**Output:** X is an array with spacing DELTA, and units of dBW. PDFX is the canonical Class A pdf of X, and N is the number of points in X and PDFX.

### 5.3.3 SUBROUTINE DTLOS (D,PL,ND,A,B,XINC,T,PT,NT,NTMAX)

This subroutine computes the pdf of transmission loss when the transmission loss is a deterministic function of a probabilistic path length.

**Input:** D is an evenly-spaced array of distances, PL is the pdf of D, and ND is the number of points in D and PL. A is the constant, and B is the growth rate in the transmission loss formula:  $\text{Loss} = A + B \log(D)$ . XINC is the spacing in the output array, T. NTMAX is the dimension of the arrays in the call.

**Output:** T is the transmission loss array, in dB, at intervals of XINC. PT is the pdf of T, and NT is the number of points at which it is defined.

### 5.3.4 SUBROUTINE EXPPDF (AV,NPTS,X,PDFX,NMAX)

This subroutine produces the dB equivalent of an exponential pdf. That is, if Y is an exponentially-distributed random variable with mean AV, then  $X = 10 \log Y$ , and PDFX is the pdf of X. The distribution is truncated and normalized.

**Input:** AV is the mean, in watts, of the exponential distribution. NPTS is the number of points at which X and PDFX will be defined. NMAX is the dimension of the arrays X and PDFX.

**Output:** X is an evenly-spaced array defined at NPTS points. Its units are dBW. PDFX is the pdf of X.

### 5.3.5 SUBROUTINE PLCON (RBEG,RINC,REND,X,WPAL,N,NMAX)

This subroutine produces a constant pdf from RBEG to REND. That is, the random variable X is uniformly-distributed in [RBEG,REND].

**Input:** RBEG is the smallest value of X and REND is the largest value of X. RINC is the increment in X. NMAX is the dimension of X and WPAL in the calling program.

**Output:** X is an evenly-spaced array ranging from RBEG to REND. WPAL is the pdf of X, and N is the number of points in X and WPAL.



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APPENDIX A: ADDITION OF RANDOM VARIABLES:  
ANALYSIS FOR RVADD AND RVSUB

Addition or subtraction of random variables requires integration of a convolution integral. This Appendix shows how the limits of integration and the integrand can be found in terms of the subscripts of the random variable array in a numerical procedure. It also shows how addition and subtraction can be handled with a single procedure.

Let  $X$  be a random variable with probability density function (pdf),  $F_X$ . (The same notation will be used to denote the random variable and the array containing it, and the pdf and the array containing the pdf.) Assume that  $X$  is an evenly spaced array such that

$$X(n) = X(1) + (n-1)D, \quad n = 1, NX, \quad (A-1)$$

and  $X(1)$  is a multiple of  $D$ . If  $X$  is not so spaced, Subroutine CHINCI (described in Section 5.1.2 of the report) will produce this spacing.

Let  $Y$  be a random variable independent of  $X$  with pdf,  $F_Y$ , and with

$$Y(m) = Y(1) + (m-1)D, \quad m=1, NY, \quad (A-2)$$

and  $Y(1)$  is a multiple of  $D$ .

Let  $Z = X + Y$ . The smallest  $Z$  will be  $Z(1) = X(1) + Y(1)$  and the largest  $Z$  will be  $Z(NZ) = X(NX) + Y(NY)$ . Let  $Z$  have the same spacing as  $X$  and  $Y$  so that

$$Z(j) = Z(1) + (j-1)D, \quad j = 1, NZ, \quad (A-3)$$

where

$$\begin{aligned} NZ &= [X(NX) + Y(NY) - Z(1)]/D + 1 \\ &= [X(1) + (NX-1)D + Y(1) + (NY-1)D - (X(1)+Y(1))]/D + 1 \\ &= NX + NY - 1. \end{aligned} \quad (A-4)$$

The pdf of Z, FZ, is found from

$$FZ(z) = \int_A^B FX(x) FY(z-x) dx, \quad (A-5)$$

where

$$A = \max[ X(1), z-Y(NY) ], \quad (A-6)$$

and

$$B = \min[ X(NX), z-Y(1) ]. \quad (A-7)$$

To find FZ numerically,

$$FZ(Z(j)) = \sum_{i=ma}^{mb} w(i) FX(X(i)) FY(Z(j) - X(i)), \quad (A-8)$$

where  $w(i)$  is a weight determined by the numerical integration formula,  $ma$  is the index such that  $X(ma) = A$ , and  $mb$  is the index such that  $X(mb) = B$ .

Now, to find A and its index:

$$\begin{aligned} Z(j) - Y(NY) &= Z(1) + (j-1)D - [Y(1) + (NY-1)D] \\ &= X(1) + Y(1) + (j-1)D - Y(1) - (NY-1)D \\ &= X(1) + (j-NY) D. \end{aligned} \quad (A-9)$$

If  $j \leq NY$ , then  $Z(j)-Y(NY) \leq X(1)$ , and  $A = X(1)$ , so  $ma = 1$ . (A-10)

But, if  $j > NY$ ,  $A = Z(j) - Y(NY) = X(1) + (j-NY+1-1)D$   
 $= X(j-NY+1)$ , so  $ma = j-NY+1$ . (A-11)

Using the same approach to finding the index mb,

$$mb = j, \text{ if } j \leq NX, \text{ and}$$

$$mb = NX, \text{ if } j > NX,$$

$$\text{that is, } mb = \min(j, NX) \tag{A-12}$$

Finally, the index my must be found such that

$$Y(my) = Z(j) - X(i), \tag{A-13}$$

because FY is known only at the points Y(m).

$$\begin{aligned} Z(j) - X(i) &= X(1) + Y(1) + (j-1)D - [X(1) + (i-1)D] \\ &= Y(1) + (j-i)D = Y(j-i+1). \end{aligned} \tag{A-14}$$

That is, my = j-i+1, so that the factor in (A-8) is

$$w(i) FX(i) FY(j-i+1), \text{ } i = ma, mb.$$

Now FZ can be computed and the only floating point arithmetic needed is that on the right side of (A-8).

Finding the difference of two random variables is similar except for the values of some of the indices. Consider  $Z = X - Y$ . The smallest value of Z now is

$$Z(1) = X(1) - Y(NY), \text{ and the largest Z is } Z(NZ) = X(NX) - Y(1), \tag{A-15}$$

$$\text{where } NZ = NX + NY - 1. \tag{A-16}$$

Now,

$$FZ(z) = \int_A^B FX(x) FY(x-z) dx, \tag{A-17}$$

where  $A = \max(X(1), Y(1)+z)$ , and  $B = \min(X(NX), Y(NY)+z)$ . (A-18)

In the numerical integration,

$$FZ(Z(j)) = \sum_{i=ma}^{mb} w(i) FX(i) FY(X(i)-Z(j)), \quad (A-19)$$

where now

$$ma = \begin{cases} 1, & \text{if } j \leq NY, \\ j-NY+1, & \text{if } j > NY, \end{cases} \quad (A-20)$$

and

$$mb = \min(NX, j). \quad (A-21)$$

Also,  $X(i) - Z(j) = Y(i-j+NY)$ . (A-22)

Comparing the results for addition and subtraction shows that both convolutions can be computed with the same procedure by defining:

$$IS = \begin{cases} +1, & \text{for addition} \\ -1, & \text{for subtraction} \end{cases} \quad (A-23)$$

$$I1 = \begin{cases} 1, & \text{for addition} \\ NY, & \text{for subtraction.} \end{cases} \quad (A-24)$$

$$I2 = \begin{cases} NY, & \text{for addition} \\ 1, & \text{for subtraction.} \end{cases} \quad (A-25)$$

Then

$$Z(1) = X(1) + IS \cdot Y(I1), \text{ and}$$

$$Z(NZ) = X(NX) + IS \cdot Y(I2). \tag{A-26}$$

Finally, (A-14) and (A-22) show that the index of FY in (A-8,-19) is

$$IS \cdot (j-i) + I1. \tag{A-27}$$

Using (A-20) through (A-27), (A-8) and (A-19) are seen to be identical. This is the procedure used in RVADD (with alternate entry RVSUB) when both X and Y are random variables.

When either X or Y is a constant, the simple linear transformation in Table 2 of the report is used.

APPENDIX B: ANALYSIS AND IMPLEMENTATION OF SUBROUTINE POWSUM

B.1 The problem

Subroutine POWSUM is an algorithm for computing the probability density function (pdf) of the power sum of two random variables given the pdfs of the two variables in decibels. More precisely, given independent random variables  $U(1) \leq U \leq U(N)$  and  $V(1) \leq V \leq V(M)$ , and their pdfs,  $f_U$  and  $f_V$ , respectively, let

$$X = 10^{U/10} \quad \text{and} \quad Y = 10^{V/10}. \quad (\text{B-1})$$

Let

$$Z = X + Y, \quad (\text{B-2})$$

and, let

$$W = 10 \log Z. \quad (\text{B-3})$$

Find  $f_W(w)$ , the pdf of  $W$ .

B.2 Analysis

Using the transformation rule in Table 2 of the report,

if  $X = 10^{U/10}$ , then the pdf of  $X$ ,  $f_X$ , is

$$f_X(x) = (C/x) f_U(10 \log x), \quad (\text{B-4})$$

where  $C = 10 \log e \approx 4.34$ . The pdf of  $Y$  is found from  $f_V$  the same way. Then

$$f_Z(z) = \int_A^B f_X(x) f_Y(z-x) dx, \quad (\text{B-5})$$

where  $A = \max(X(1), z-Y(M))$ , and  $B = \min(X(N), z-Y(1))$ . (B-6)



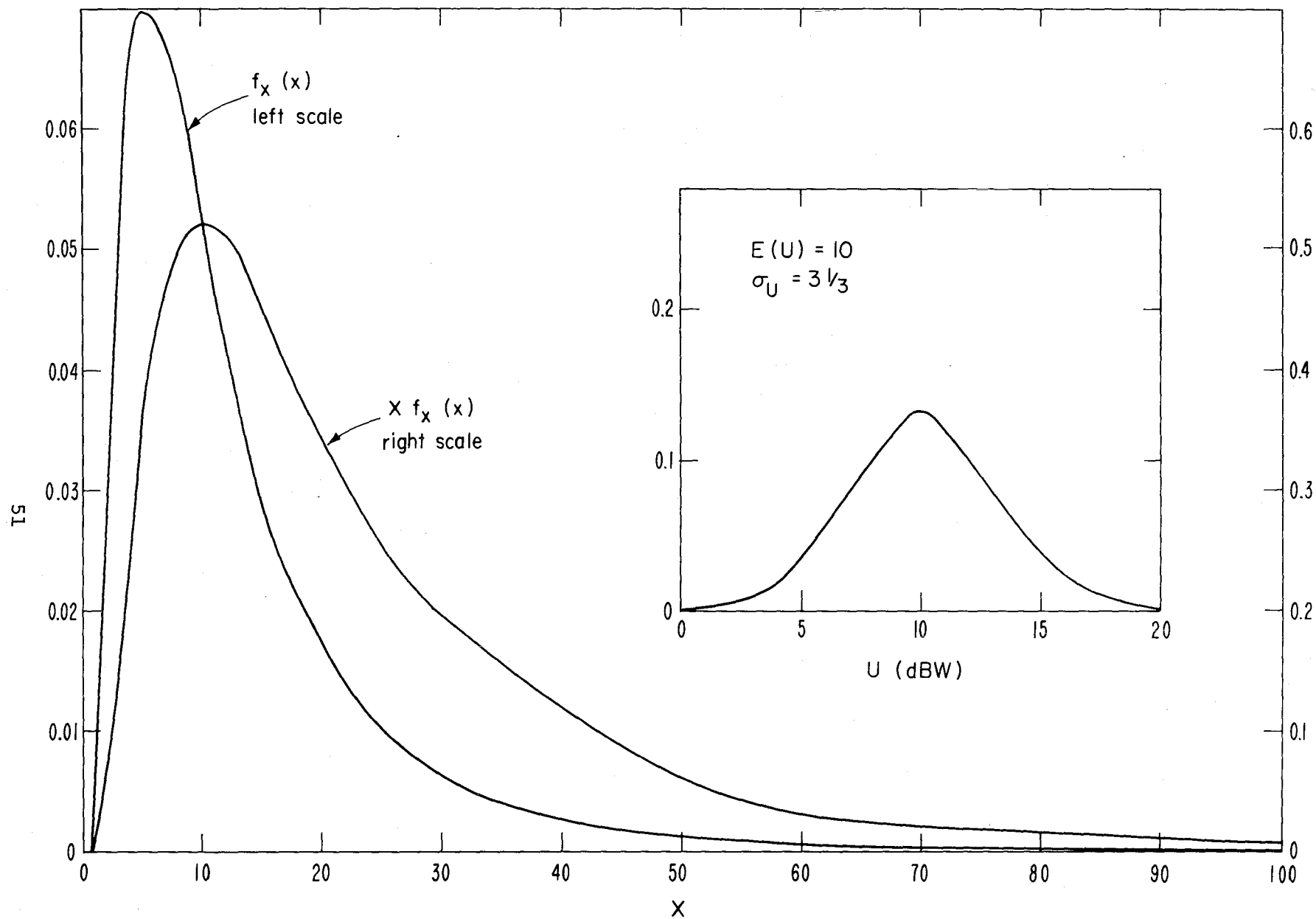


Figure B.1. Probability density function of  $X = 10 \log U$ , where the normal pdf of  $U$  is shown in the inset. Also shown is  $X$  times the pdf of  $X$  (right scale).

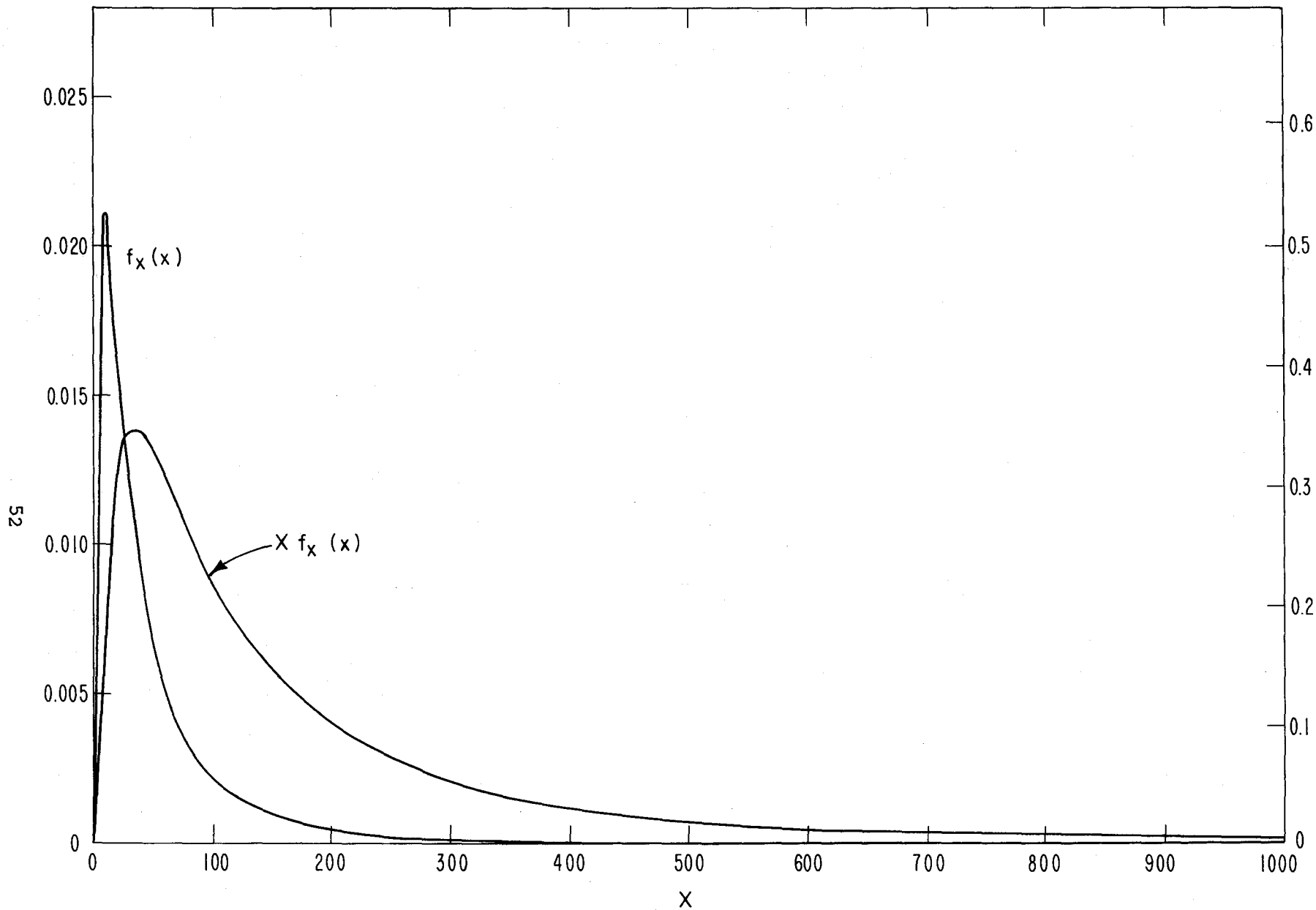


Figure B.2. Probability density function of  $X$  and of  $X$  times the pdf, where  $X$  is the dB equivalent of a normally distributed variable with a mean of 15 and a standard deviation of 5.

For reasons that will be given in the section on implementation, it is useful to rewrite (B-5) as

$$f_Z(z) = \int_A^B f_X(x)f_Y(z-x)dx = \int_A^D f_X(x)f_Y(z-x)dx + \int_D^B f_X(x)f_Y(z-x)dx. \quad (B-7)$$

Let  $s = z-x$ , and transform the integral from D to B so that  $s$  is the variable of integration. Then

$$f_Z(z) = \int_A^D f_X(x)f_Y(z-x)dx + \int_{Z-B}^{Z-D} f_Y(s)f_X(z-s)ds. \quad (B-8)$$

Finally, to convert the sum back to dB, let  $W = 10 \log Z$ . Then

$$f_W(w) = (1/C) 10^{W/10} f_Z(10^{W/10}). \quad (B-9)$$

## B.2 Numerical analysis and implementation

### B.2.1 Motivation for numerical integration method

The transformation (B-4) distorts the pdf so that standard numerical integration methods are inadequate--or, at least, inefficient. The problem will be illustrated with an example.

The inset of Figure B.1 shows a representative pdf of  $U$ . In this case,  $U$  is normally-distributed with a mean of 10 dBW and a standard deviation of 3.33 dB. The main portion of Figure B.1 shows the transformed pdf,  $f_X(x)$ , which has a sharp peak to the left and a long tail to the right. Only about 2/3 of the points used to define  $f_U(u)$  are involved in the significant portion of  $f_X(x)$ . This stretching of the distance between tabular points increases the error in subsequent numerical integration.

Figure B.1 is, in fact, an atypically "nice" case. The distortion of the pdf under the transformation is greater for larger standard deviations. Figure B.2 shows  $f_X(x)$  for  $U$  normal with a mean of 20 dBW and standard deviation of 5 dB. Many of the distributions in radio interference calculations have standard deviations of 8-12 dB (for example, atmospheric or manmade noise, and transmission loss). When

these pdfs are transformed,  $fX(x)$  looks like a right angle, with vertex in the lower left corner of Figure B.2. The pdf  $fY(y)$  in (B-5) will have a similar shape. When two such "spikey" functions are convolved, numerical integration must be done very carefully.

One solution would be to use a standard adaptive numerical integration routine. Such a routine would insert many additional integration points in regions where the integrand is changing rapidly. These points would have to be found by interpolation (because the pdfs are tabular rather than analytical), so the accuracy of the integration would be no better than that of the interpolation. Accuracy of integration without the additional points would depend on that same accuracy of interpolation, so that inserting the additional points would not produce a more accurate answer. It is better to take advantage of the known features of the special pdfs involved.

#### B.2.2 Numerical integration

Part of the peakiness of the transformed pdfs is caused by dividing by  $x$  in (B-4). This is illustrated in Figures B.1 and B.2 by the curve labeled  $X f_x(x)$ . The significant probability (area under the curve) is distributed over more of the range of  $x$  for this curve, and its maximum slope is smaller. Taking advantage of these two features will increase the accuracy of numerical integration, but will require a special integration formula.

Let

$$g(x) = C fU(10 \log x) \quad (B-10)$$

and

$$h(y) = C fV(10 \log y). \quad (B-11)$$

Using (B-4), (B-10) and (B-11) in (B-5) produces

$$fZ(z) = \int_A^B [g(x)/x] [h(z-x)/(z-x)] dx \quad (B-12)$$

with a similar change in (B-8).

The reason for changing the variable of integration over part of its range to get (B-8) will now be given. Suppose two random variables, U and V, each with the distribution shown in the inset in Figure B.1, are to be transformed to power and added. Let  $M = N = 15$ . Table B.1 shows U in dBW and its pdf in columns 2 and 3. Column 4 of Table B.1 shows the corresponding X computed using (B-4), column 5 shows the pdf of x,  $f_X(x)$ , and column 6 shows  $xf_X(x)$ . Since V has the same distribution as U, Table B.1 can be used for U and Y and their pdfs. The pdf  $f_Z(z(i))$  is to be computed for  $z(i) = x(i) + y(i)$ .

First, consider  $Z(5) = x(5) + y(5) = 7.456$ . The limits A and B are found using (B-6);

$$A = \max[x(1)=1, z(5)-y(15)=-92.544] = x(1), \text{ and}$$

$$B = \min[y(15)=100, z(5)-y(1)=6.456] = 6.456.$$

The integration can be done using (B-5) with the integration points listed in Table B.2. The points  $z(5) - x(i)$  shown in Table B.2 are an adequate sample of the function  $h(y)$  over the range of integration. That is,  $h(y)$  is smooth and monotonic between sample points and interpolation between the points yields a sufficiently-accurate approximation of  $h(y)$  between  $z(5) - A$  and  $z(5) - B$ .

In contrast, consider  $z(13) = 103.58$ . Now

$$A = \max[x(1)=1, z(13)-y(15)=3.58] = 3.58, \text{ and}$$

$$B = \min[x(15)=100, z(13)-y(1)=102.58] = 100.$$

If (B-5) is used for the integration, the sample points are those given in Table B.3. Look at where the last two values of  $z-x$  fall on Figure B.1 (or in Table B.1). The function  $h(z-x)$  [which is  $yf_Y(z-y)$  in Table B.1, see (B-11)] is not adequately sampled by the values of  $z-x$  in Table B.3. The main peak of the pdf is missed. If the values of  $xf_X(x)$  were plotted at just the points  $z-x$  in Table B.3, one would think that the peak of the function were near 32, and had a value of about 0.2. But the peak is actually at 10 with a value of 0.52. So if (B-5) is used to find  $f_Z(z(13))$ , more sample points must be inserted between  $x(14)$  and  $x(15)$ . How

Table B.1 A Random Variable, U, in Decibels, and its Probability Density Function fU Converted to Power X = 10\*\*(U/10) and the Probability Density Function of X, fX.

i	U	fU	X	fX	X*fX
1	0.00	0.0013	1.000E+00	5.789E-03	5.791E-03
2	1.43	0.0044	1.390E+00	1.375E-02	1.911E-03
3	2.86	0.0121	1.931E+00	2.717E-02	5.247E-02
4	4.29	0.0276	2.683E+00	4.470E-02	1.199E-01
5	5.71	0.0525	3.728E+00	6.118E-02	2.281E-01
6	7.14	0.0831	5.180E+00	6.969E-02	3.610E-01
7	8.57	0.1095	7.197E+00	6.607E-02	4.755E-01
8	10.00	0.1200	1.000E+01	5.212E-02	5.212E-01
9	11.43	0.1095	1.389E+01	3.422E-02	4.755E-01
10	12.86	0.0831	1.931E+01	1.870E-02	3.610E-01
11	14.29	0.0525	2.682E+01	8.503E-03	2.281E-01
12	15.71	0.0276	3.727E+01	3.218E-03	1.199E-01
13	17.14	0.0121	5.179E+01	1.013E-03	5.247E-02
14	18.57	0.0044	7.195E+01	2.656E-04	1.911E-02
15	20.00	0.0013	1.000E+02	5.792E-05	5.791E-03

Table B.2 X and z - X for Integration when z = 7.456.

j	X	z - X
1	1.000	6.456
2	1.390	6.066
3	1.931	5.525
4	2.683	4.773
5	3.728	3.728
6	5.180	2.276
B	6.456	1.000

Table B.3 X and z - X for Integration when z = 103.58.

j	X	z - X
A	3.580	100.00
6	5.180	98.40
7	7.197	96.38
8	10.000	93.58
9	13.89	89.69
10	19.31	84.27
11	26.82	76.76
12	37.27	66.31
13	51.79	51.79
14	71.95	31.63
15	100.00	3.58

many should be inserted, and where should they be to take maximum advantage of the information in Table B.1? The answer comes from the rightmost integral in (B-8). If the points  $y(i)$  are chosen as the sample points for  $s$  in (B-8) no interpolation is needed and we get the best possible values for the function  $h(y)$ .

So define  $D = x(14) = 71.95$  in (B-8). Then  $Z-B = 3.58$ ,  $Z-A = 100$ , and the integration points in the rightmost integral in (B-8) are the same as those in Table B.3 with  $y$  written in place of  $x$ . Now the integration of  $fZ(z)$  using (B-8) uses all possible samples of  $g(x)$  and  $h(y)$  with the fewest possible interpolations in the input tables. The use of the input information has been optimized.

The idea above is implemented as follows: Use (B-8) with input points  $y(i)$  as the variable of integration in the rightmost integral whenever this would result in sampling  $h(y)$  more often than use of (B-5).

The idea of choosing the variable of integration to optimize use of the input information is extended to other considerations. Implementation of both (B-5) and (B-8) is easier if both  $U$  and  $V$  are defined at the same number of points. But if  $V$  (say) has a much smaller range than  $U$ , then the  $y$  points will be much closer together than the  $x$  points and the most samples of the functions will result from using  $y$  as the variable of integration in (B-5). In this case, the roles of  $x$  and  $y$  in (B-5) and (B-8) are interchanged (notice that the equations are symmetric in  $x$  and  $y$ ). The same thing may occur if the mean of  $V$  is much smaller than the mean of  $U$  even if they have the same range. In general, the primary variable of integration [the variable of integration in (B-5) or in the left integral in (B-8)] is chosen to be the one that results in the most samples of the functions in the integrand, and then (B-8) is used whenever its use will increase the number of samples.

### B.2.3 Implementation

Subroutine POWSUM implements the procedure described in this appendix. First, it ensures that both variables  $U$  and  $V$  are defined at the same number of points, and then converts them to  $X$  and  $Y$  using (B-3). The functions  $g(x)$  and  $h(y)$  are computed using (B-4), (B-10), and (B-11).

Subroutine GO computes the limits of integration for the integral in (B-5) and the number of points that would be used in a numerical integration. Then POWSUM makes the tests described in the last two paragraphs of the previous section, and chooses the primary variable of integration, and decides whether (B-5) or (B-8) will be used. The numerical integration is done in Function SPINT.

## APPENDIX C: TROUBLE REPORTS

This appendix lists the trouble reports that may be printed by the program, and suggestions for avoiding the trouble. However, some problems will require more extensive or creative fixes that the user can determine only by studying the program listing and the analysis in Section 4.

The trouble reports are listed alphabetically by beginning word.

**A OUTSIDE RANGE IN INTERP.** A,X(1),X(2),X(NX-1),X(NX) = ( 5 numbers in E format)

The interpolation subroutine is being asked to extrapolate outside the range of the input table. Check input variables for blunders. For example, is PAR(10), the maximum wanted path length, less than PAR(8), the minimum wanted path length?

**DIMENSION TOO LARGE IN RVADD, NA,NB,NCMAX=** (three integers).  
**CALLED FROM RVADD (or RVSUB).**

Try doubling PAR(116), the dB increment in pdfs.

**IMAX TOO SMALL IN CHINC.** N,IMAX,X(1),X(NX),D,DEL = (five numbers)

An attempt has been made to exceed the storage space for an array. The numbers may help identify the particular array involved. Increase the increment size or decrease the number of points in that array. Or, as a possible quick fix, increase the dB increment in the pdfs, PAR(116), the increment in wanted path length, PAR(9), or the increment in unwanted path length, PAR(63).

**IN PLAN I = (integer) IS GREATER THAN NMAX = (integer) INCREASE RINC**  
Increase the increment in unwanted path length, PAR(63).

**IN QUAD1, N IS NEGATIVE OR ZERO, N = (integer)**

Look for absurd input, such as a "maximum" that is less than a "minimum."

**IN RENORME, ONE = (number)**

The integral of a pdf is not approximately one. Look for absurd input. Or, if you have modified the program, make sure all pdfs are proper pdfs; that is, that the integral of the pdf over its range is one.

**IN RENORMR, SUM = (number)**

Same as previous comment.

**IN SHRINK, N > NYMAX. NYMAX = (integer)**

Increase the path length increments, PAR(9) and PAR(63), or the dB increment, PAR(116).

**IN TLOS NT = (integer) IS LARGER THAN NTMAX = (integer) INCREASE XINC**  
Increase PAR(116).



IN TOTPD, NZT = (integer) GREATER THAN NZMAX = (integer)  
Increase PAR(116). Or perhaps the mean noise, PAR(101),  
is too much smaller than the mean interference, which is  
controlled (among other things), by the mean of the  
unwanted transmitter ERP, PAR(51).

IN WPLRW, N IS TOO LARGE, N = (integer)  
Increase the increment in wanted path length, PAR(9).

NW EXCEEDS DIMENSION OF W IN POWSUM. NW, NWTOP, N1, N, Z(1), Z(NZ), DW = (5 numbers)  
Increase the dB increment, PAR(116).

NX = (integer) IS GREATER THAN NMAX = (integer) IN AVSIG  
Increase the dB increment, PAR(116).

NX .GT. IMAX IN CHINC. NX, IMAX = (two integers)  
CALLED WITH ENTRY CHINC (or CHINCI)  
Increase one or more of the increments, PAR(9), PAR(63),  
and PAR(116). Or look for absurdly large ranges in  
path length.

NX2 = (integer) AND DIMENSION IS (integer) IN SPINT  
Increase the dB increment, PAR(116).

WARNING: R.V. INPUT TO POWSUM < -193

TRY CHANGING INPUT UNITS (E.G. DBW TO DBM OR DBKW)

The trouble occurs in the minicomputer version. To avoid underflows  
change input units of power (means of wanted and unwanted transmitter  
power and mean of ambient noise) to DBM by adding 60 to the values in  
DBW. Remember to interpret the output in DBM.

WARNING: R.V. INPUT TO POWSUM > 190

TRY CHANGING INPUT UNITS (E.G. DBW TO DBM OR DBKW)

Prevents overflow in minicomputer version. Change units of noise and  
wanted and unwanted transmitter powers to DBKW by subtracting 30 from  
each. Interpret output in DBKW.



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1. PUBLICATION NO. NTIA Report 82-106		2. Gov't Accession No.	3. Recipient's Accession No.
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15. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.) PRODSIR is a procedure for computing the <u>probability</u> distribution of the <u>signal-to-interference</u> ratio in a congested radio environment. Most of the independent variables in the usual signal-to-interference equation are assumed to be random variables, and the input to the program is their probability density functions (pdfs). The pdfs can be analytical or tabular. The times and lengths of interfering emissions are assumed to be random. PRODSIR combines and transforms the input pdfs using the relationships of probability theory and produces the pdfs of the wanted signal power, the interference-plus-noise power, and the probability distribution of the signal-to-interference ratio. PRODSIR is implemented in an interactive program. A sample session for a version of PRODSIR involving only cochannel interference and noise is included to illustrate the input and output. The structure of the program is given along with instructions for modifying it for special applications. Appendices describe some of the subroutines for handling random variables.			
16. Key Words (Alphabetical order, separated by semicolons) Interference; probabilistic EMC; signal-to-interference ratio; spectrum management.			
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